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ON GENERAL CONFORMAL METRICAL N-LINEAR CONNECTIONS ON DUAL BUNDLE OF K-TANGENT BUNDLE

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Abstract: *In the present paper we give the transformations for the coefficients of an N-linear connection on dual bundle of k-tangent bundle,* $T^{*k}M$, *by a transformation of a nonlinear connection on* $T^{*k}M$, $k \geq 2, k \in N$. *Starting from the notion of conformal metrical d-structure we define the notion of general conformal metrical N-linear connection on dual bundle of k-tangent bundle.We determine the set of all general conformal metrical N-linear connections,in the case when the nonlinear connection is fixed and we find important particular cases. Finally we find the transformations group of these connections.*

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1. INTRODUCTION

The notion of Hamilton space was introduced by Acad. R. Miron in [5],[6].The Hamilton spaces appear as dual via Legendre transformation, of the Lagrange spaces.

The differential geometry of the dual bundle of $k-$osculator bundle was introduced and studied by Acad. R. Miron [10].

In the present section we keep the general setting from Acad. R. Miron $[10]$, and subsequently we recall only some needed notions. For more details see $[10]$.

Let M be a real n-dimensional $C^{\infty}-$manifold and let $\left(T^{*k}M, \pi^{*k}, M\right)$, $\left(k \geq 2, k \in N\right)$ be the dual bundle of k-osculator bundle (or k-cotangent bundle), where the total space is

$$T^{*k}M = T^{*k-1}M \times T^{*}M. \qquad (1)$$

Let $\left(x^{i}, y^{(1)i}, \ldots, y^{(k-1)i}, p_{i}\right), (i = 1,2,\ldots,n)$, be the local coordinates of a point $u = \left(x, y^{(1)}, \ldots, y^{(k-1)}, p\right) \in T^{*k}M$ in a local chart on $T^{*k}M$.The change of coordinates on the manifold $T^{*k}M$ *is*

$$\begin{cases}\widetilde{x}^i=\widetilde{x}^i\left(x^1,...,x^n\right), \det\left(\dfrac{\partial\widetilde{x}^i}{\partial x^j}\right)\neq 0,\\ \widetilde{y}^{(1)i}=\dfrac{\partial\widetilde{x}^i}{\partial x^j}y^{(1)j},\\ \dots\dots\dots\dots\\ (k-1)\widetilde{y}^{(k-1)i}=\dfrac{\partial\widetilde{y}^{(k-2)i}}{\partial x^j}y^{(1)j}+...+(k-1)\dfrac{\partial\widetilde{y}^{(k-2)i}}{\partial y^{(k-2)j}}y^{(k-1)j}\\ \widetilde{p}_i=\dfrac{\partial x^j}{\partial\widetilde{x}^i}p_j,\end{cases} \tag{2}$$

We denote with N a nonlinear connection on the manifold $T^{*k}M$, with the coefficients

$$\left(\underset{(1)}{N}{}^j{}_i\left(x,y^{(1)},...,y^{(k-1)},p\right),...,\underset{(k-1)}{N}{}^j{}_i\left(x,y^{(1)},...,y^{(k-1)},p\right)\right. \tag{3}$$
$$\left.N_{ij}\left(x,y^{(1)},...,y^{(k-1)},p\right)\right),(i,j=1,2,...,n).$$

The tangent space of $T^{*k}M$ in the point $u\in T^{*k}M$ is given by the direct sum of vector spaces

$$T_u\left(T^{*k}M\right)=N_{0,u}\oplus N_{1,u}\oplus...\oplus\oplus N_{k-2,u}\oplus V_{k-1,u}\oplus W_{k,u},\forall u\in T^{*k}M \tag{4}$$

A local adapted basis to the direct decomposition (4) is given by

$$\left\{\frac{\delta}{\delta x^i},\frac{\delta}{\delta y^{(1)i}},...,\frac{\delta}{\delta y^{(k-1)i}},\frac{\delta}{\delta p_i}\right\},(i=1,2,...,n), \tag{5}$$

where

$$\begin{cases}\dfrac{\delta}{\delta x^i}=\dfrac{\partial}{\partial x^i}-\underset{(1)}{N}{}^j{}_i\dfrac{\partial}{\partial y^{(1)j}}-...-\underset{(k-1)}{N}{}^j{}_i\dfrac{\partial}{\partial y^{(k-1)j}}+N_{ij}\dfrac{\partial}{\partial p_j}\\ \dfrac{\delta}{\delta y^{(1)i}}=\dfrac{\partial}{\partial y^{(1)}{}_i}-\underset{(1)}{N}{}^j{}_i\dfrac{\partial}{\partial y^{(2)j}}-...-\underset{(k-2)}{N}{}^j{}_i\dfrac{\partial}{\partial y^{(k-1)j}}\\ \dots\dots\dots\dots\\ \dfrac{\delta}{\delta y^{(k-1)i}}=\dfrac{\partial}{\partial y^{(k-1)i}},\\ \dfrac{\delta}{\delta p_i}=\dfrac{\partial}{\partial p_i}.\end{cases} \tag{6}$$

Let D be an $N-$linear connection on $T^{*k}M$, with the local coefficients in the adapted basis

$$D\Gamma(N)=\left(H^i{}_{jh},\underset{(\alpha)}{C}{}^i{}_{jh},C_i{}^{jh}\right),(\alpha=1,...,k-1) \tag{7}$$

2.THE SET OF THE TRANSFORMATIONS OF $N-$LINEAR CONNECTIONS

Let $\overline{N}$ be another nonlinear connection on $T^{*k}M$, with the local coefficients

$$\left(\underset{(1)}{\overline{N}}{}^j{}_i\left(x,y^{(1)},...,y^{(k-1)},p\right),...,\underset{(k-1)}{\overline{N}}{}^j{}_i\left(x,y^{(1)},...,y^{(k-1)},p\right),\right.$$
$$\left.N_{ij}\left(x,y^{(1)},...,y^{(k-1)},p\right)\right)(i,j=1,2,...,n).$$

Then there exists the uniquely determined tensor fields $\underset{(\alpha)}{A}{}^j{}_i\in\tau_1^1\left(T^{*k}M\right)$, $(\alpha=1,...,k-1)$ and $A_{ij}\in\tau_2^0\left(T^{*k}M\right)$, such that

$$\begin{cases}\underset{(\alpha)}{\overline{N}}{}^i{}_j=\underset{(\alpha)}{N}{}^i{}_j-\underset{(\alpha)}{A}{}^i{}_j,(\alpha=1,...,k-1),\\ \overline{N}_{ij}=N_{ij}-A_{ij},(i,j=1,2,...,n).\end{cases} \tag{8}$$

Conversely, if $\underset{(\alpha)}{N}{}^i{}_j$ and $\underset{(\alpha)}{A}{}^i{}_j,(\alpha=1,...,k-1)$, respectively N_{ij} and A_{ij} are given, then $\underset{(\alpha)}{\overline{N}}{}^i{}_j,(\alpha=1,...,k-1)$, respectively $\overline{N}_{ij}$, given by (8) are the coefficients of a nonlinear connection.

Theorem 1 *Let N and $\overline{N}$ be two nonlinear connections on $T^{*k}M,(k\geq 2,\ k\in N)$, with local coefficients*

$$\left(\underset{(1)}{N}{}^j{}_i\left(x,y^{(1)},...,y^{(k-1)},p\right),...,\underset{(k-1)}{N}{}^j{}_i\left(x,y^{(1)},...,y^{(k-1)},p\right)\right.$$
$$\left.N_{ij}\left(x,y^{(1)},...,y^{(k-1)},p\right)\right),\left(\underset{(1)}{\overline{N}}{}^j{}_i\left(x,y^{(1)},...,y^{(k-1)},p\right),\dots,\right.$$
$$\left.\underset{(k-1)}{\overline{N}}{}^j{}_i\left(x,y^{(1)},...,y^{(k-1)},p\right),\overline{N}_{ij}\left(x,y^{(1)},...,y^{(k-1)},p\right)\right)$$

$i,j=1,2,...,n,$ respectively.

If $D\Gamma(N)=\left(H^i{}_{jh},\underset{(\alpha)}{C}{}^i{}_{jh},C_i{}^{jh}\right)$ and

$$D\overline{\Gamma}\left(\overline{N}\right)=\left(\overline{H}^i{}_{jh},\underset{(\alpha)}{\overline{C}}{}^i{}_{jh},\overline{C}_i{}^{jh}\right),\ (\alpha=1,...,k-1)$$

are the local coefficients of two $N-$, respectively $\overline{N}$-linear connections, D, respectively $\overline{D}$ on the differentiable manifold $T^{*k}M,(k\geq 2,k\in N)$, then the transformation

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$N \to \overline{N}$, given by (8) of nonlinear connections implies for the coefficients of the $\overline{N}$-linear connection $D\Gamma(\overline{N}) = \left(\overline{H}^i{}_{jh}, \underset{(\alpha)}{\overline{C}}{}^i{}_{jh}, \overline{C}_i{}^{jh}\right)$, the following relations

$$\begin{aligned}
\overline{H}^i{}_{sj} &= H^i{}_{sj} + \underset{(1)}{A}{}^m{}_j \Big[\underset{(1)}{C}{}^i{}_{sm} + \underset{(1)}{N}{}^l{}_m \underset{(2)}{C}{}^i{}_{sl} + \ldots + \underset{(k-2)}{N}{}^l{}_m \underset{(k-1)}{C}{}^i{}_{sl} + \underset{(1)}{N}{}^l{}_m \underset{(1)}{N}{}^t{}_l \underset{(3)}{C}{}^i{}_{st} + \ldots + \\
&\left(\underset{(1)}{N}{}^l{}_m \underset{(k-3)}{N}{}^t{}_l + \ldots + \underset{(k-3)}{N}{}^l{}_m \underset{(1)}{N}{}^t{}_l \right) \underset{(k-1)}{C}{}^i{}_{st} + \ldots + \underbrace{\underset{(1)}{N}\ldots\underset{(1)}{N}}_{(k-2)} \underset{(k-1)}{C} \Big] + \qquad (9)\\
&+ \underset{(2)}{A}{}^m{}_j \Big[\underset{(2)}{C}{}^i{}_{sm} + \underset{(1)}{N}{}^l{}_m \underset{(3)}{C}{}^i{}_{sl} + \ldots + \underset{(k-3)}{N}{}^l{}_m \underset{(k-1)}{C}{}^i{}_{st} + \ldots + \underbrace{\underset{(1)}{N}\ldots\underset{(1)}{N}}_{(k-3)} \underset{(k-1)}{C} \Big] + \\
&+ \ldots + \underset{(k-2)}{A}{}^m{}_j \left(\underset{(k-2)}{C}{}^i{}_{sm} + \underset{(1)}{N}{}^l{}_m \underset{(k-1)}{C}{}^i{}_{sl} \right) + \\
&+ \underset{(k-1)}{A}{}^m{}_j \underset{(k-1)}{C}{}^i{}_{sm} - A_{jm} C_s{}^{im}, \\
\underset{(1)}{\overline{C}}{}^i{}_{sj} &= \underset{(1)}{C}{}^i{}_{sj} + \underset{(1)}{A}{}^m{}_j \Big[\underset{(2)}{C}{}^i{}_{sm} + \underset{(1)}{N}{}^r{}_m \underset{(3)}{C}{}^i{}_{sr} + \ldots + \\
&+ \underset{(k-3)}{N}{}^r{}_m \underset{(k-1)}{C}{}^i{}_{sr} + \ldots + \underbrace{\underset{(1)}{N}\cdot\ldots\cdot\underset{(1)}{N}}_{(k-3)} \underset{(k-1)}{C} \Big] + \\
&+ \ldots + \underset{(k-3)}{A}{}^m{}_j \left[\underset{(k-2)}{C}{}^i{}_{sm} + \underset{(1)}{N}{}^r{}_m \underset{(k-1)}{C}{}^i{}_{sr} \right] + \\
&+ \underset{(k-2)}{A}{}^m{}_j \underset{(k-1)}{C}{}^i{}_{sm}, \quad \underset{(k-2)}{\overline{C}}{}^i{}_{sj} = \underset{(k-2)}{C}{}^i{}_{sj} + \underset{(1)}{A}{}^l{}_j \underset{(k-1)}{C}{}^i{}_{sl}, \\
\underset{(k-1)}{\overline{C}}{}^i{}_{sj} &= \underset{(k-1)}{C}{}^i{}_{sj}, \quad \overline{C}_s{}^{ij} = C_s^{ij},
\end{aligned}$$

$$\begin{cases} \underset{(1)}{A}{}^h{}_{i|j} = 0, \\ A_{ih|j} = 0, (i,j,h = 1,2\ldots,n), \end{cases}$$

where " $|$ " denotes the $h-$covariant derivative with respect to $D\Gamma(N)$.

Theorem 2 *Let N and $\overline{N}$ be two nonlinear connections on $T^{*k}M, (k \geq 2,\ k \in N)$, with local coefficients*

$$\left(\underset{(1)}{N}{}^j{}_i\left(x, y^{(1)}, \ldots, y^{(k-1)}, p\right), \ldots, \underset{(k-1)}{N}{}^j{}_i\left(x, y^{(1)}, \ldots, y^{(k-1)}, p\right), N_{ij}\left(x, y^{(1)}, \ldots, y^{(k-1)}, p\right) \right), \left(\underset{(1)}{\overline{N}}{}^j{}_i\left(x, y^{(1)}, \ldots, y^{(k-1)}, p\right), \ldots, \underset{(k-1)}{\overline{N}}{}^j{}_i\left(x, y^{(1)}, \ldots, y^{(k-1)}, p\right), \overline{N}_{ij}\left(x, y^{(1)}, \ldots, y^{(k-1)}, p\right) \right)$$

$i, j = 1,2,\ldots,n,$ respectively.

If $D\Gamma(N) = \left(H^i{}_{jh}, \underset{(\alpha)}{C}{}^i{}_{jh}, C_i{}^{jh} \right)$ and

$$D\overline{\Gamma}(\overline{N}) = \left(\overline{H}^i{}_{jh}, \underset{(\alpha)}{\overline{C}}{}^i{}_{jh}, \overline{C}_i{}^{jh} \right), \qquad (\alpha = 1,\ldots,k-1)$$

are the local coefficients of two $N-$, respectively $\overline{N}$-linear connections, D, respectively $\overline{D}$ on the differentiable manifold $T^{*k}M, (k \geq 2, k \in N)$, then there exists only one system of tensor fields

$$\left(\underset{(1)}{A}^i{}_j,\dots,\underset{(k-1)}{A}^i{}_j, A_{ij}, B^i{}_{jh}, \underset{(1)}{D}^i{}_{jh},\dots,\underset{(k-1)}{D}^i{}_{jh}, D_i{}^{jh}\right)$$

such that

$$\underset{(\alpha)}{\overline{N}}^i{}_j = \underset{(\alpha)}{N}^i{}_j - \underset{(\alpha)}{A}^i{}_j, (\alpha = 1,\dots,k-1),$$

$$\overline{N}_{ij} = N_{ij} - A_{ij},$$

$$\begin{aligned}
\overline{H}^i{}_{sj} = H^i{}_{sj} &+ \underset{(1)}{A}^m{}_j \Big[\underset{(1)}{C}^i{}_{sm} + \underset{(1)}{N}^l{}_m \underset{(2)}{C}^i{}_{sl} + \dots + \\
&+ \underset{(k-2)}{N}^l{}_m \underset{(k-1)}{C}^i{}_{sl} + \underset{(1)}{N}^l{}_m \underset{(1)}{N}^t{}_l \underset{(3)}{C}^i{}_{st} + \dots + \\
&\left(\underset{(1)}{N}^l{}_m \underset{(k-3)}{N}^t{}_l + \dots + \underset{(k-3)}{N}^l{}_m \underset{(1)}{N}^t{}_l\right) \underset{(k-1)}{C}^i{}_{st} + \dots \\
&+ \underbrace{\underset{(1)}{N}\dots\underset{(1)}{N}}_{(k-2)} \underset{(k-1)}{C} \Big] + \qquad (10) \\
&+ \underset{(2)}{A}^m{}_j \Big[\underset{(2)}{C}^i{}_{sm} + \underset{(1)}{N}^l{}_m \underset{(3)}{C}^i{}_{sl} + \dots + \underset{(k-3)}{N}^l{}_m \underset{(k-1)}{C}^i{}_{st} + \dots + \underbrace{\underset{(1)}{N}\dots\underset{(1)}{N}}_{(k-3)} \underset{(k-1)}{C} \Big] + \\
&+ \dots + \underset{(k-2)}{A}^m{}_j \left(\underset{(k-2)}{C}^i{}_{sm} + \underset{(1)}{N}^l{}_m \underset{(k-1)}{C}^i{}_{sl} \right) + \\
&+ \underset{(k-1)}{A}^m{}_j \underset{(k-1)}{C}^i{}_{sm} - A_{jm} C_s{}^{im} - B^i{}_{sj},
\end{aligned}$$

$$\begin{aligned}
\underset{(1)}{\overline{C}}^i{}_{sj} = \underset{(1)}{C}^i{}_{sj} &+ \underset{(1)}{A}^m{}_j \Big[\underset{(2)}{C}^i{}_{sm} + \underset{(1)}{N}^r{}_m \underset{(3)}{C}^i{}_{sr} + \dots + \\
&+ \underset{(k-3)}{N}^r{}_m \underset{(k-1)}{C}^i{}_{sr} + \dots + \underbrace{\underset{(1)}{N}\cdot\dots\cdot\underset{(1)}{N}}_{(k-3)} \underset{(k-1)}{C} \Big] + \\
&+ \dots + \underset{(k-3)}{A}^m{}_j \Big[\underset{(k-2)}{C}^i{}_{sm} + \underset{(1)}{N}^r{}_m \underset{(k-1)}{C}^i{}_{sr} \Big] + \\
&+ \underset{(k-2)}{A}^m{}_j \underset{(k-1)}{C}^i{}_{sm} - \underset{(1)}{D}^i{}_{sj},
\end{aligned}$$

$$\underset{(k-2)}{\overline{C}}^i{}_{sj} = \underset{(k-2)}{C}^i{}_{sj} + \underset{(1)}{A}^l{}_j \underset{(k-1)}{C}^i{}_{sl} - \underset{(k-2)}{D}^i{}_{sj},$$

$$\underset{(k-1)}{\overline{C}}^i{}_{sj} = \underset{(k-1)}{C}^i{}_{sj} - \underset{(k-1)}{D}^i{}_{sj}, \overline{C}_s{}^{ij} = C_s^{ij} - D_s{}^{ij},$$

$$\begin{cases} \underset{(1)}{A}^h{}_{i|j} = 0, \\ A_{ih|j} = 0, (i,j,h = 1,2\dots,n), \end{cases}$$

where " | " denotes the $h-$covariant derivative with respect to $D\Gamma(N)$.

In the particular case when we have the same nonlinear connection N, that is $\underset{(\alpha)}{A}^i{}_j = 0$, $(\alpha = 1,\dots,k-1), (k \geq 2, k \in N)$ and $A_{ij} = 0$, we obtain the set of transformations of N-linear connections corresponding to the same nonlinear connection N given by

$$\begin{cases} \overline{H}^i{}_{sj} = H^i{}_{sj} - B^i{}_{sj}, \\ \underset{(\alpha)}{\overline{C}}^i{}_{sj} = \underset{(\alpha)}{C}^i{}_{sj} - \underset{(\alpha)}{D}^i{}_{sj}, (\alpha = 1,\dots,k-1), (k \geq 2, k \in N) \\ \overline{C}_s{}^{ij} = C_s^{ij} - D_s^{ij}. \end{cases} \quad (11)$$

3. GENERAL CONFORMAL METRICAL N-LINEAR CONNECTIONS IN THE HAMILTON SPACE OF ORDER $k, k \geq 2, k \in N$

Let $H^{(k)n} = (M,H)$ be a Hamiltonian space of order $k, k \geq 2, k \in N$, and let N be the cannonical nonlinear connection of the space $H^{(k)n}$ ([10], p.192).

We consider the adapted basis $\left\{\frac{\delta}{\delta x^i}, \frac{\delta}{\delta y^{(1)i}}, \frac{\delta}{\delta y^{(k-1)i}}, \frac{\delta}{\delta p_i}\right\}$ and its dual basis $\{\delta x^i, \delta y^{(1)i}, \ \delta y^{(k-1)i}, \delta p_i\}$ determined by N and by the distribution W_k. Let

$$g^{ij}(x, y(1),\dots, y^{(k-1)}, p) = \frac{1}{2}\frac{\partial^2 H}{\partial p_i \partial p_j} \qquad (12)$$

be the fundamental tensor of the space $H^{(k)n}$ [10].

The d-tensor field g^{ij} being nonsingular on $\widetilde{T^{*k}M} = T^{*k}M - \{0\}$ (where 0 is the nul section of the projection π^{*k}) there exists a d-tensor field g_{ij} covariant of order 2, symmetric,

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uniquely determined, at every point $u \in \widetilde{T^{*k}M}$, by $g_{ij}g^{jk} = \delta_i^k$ (13)

Definition 1 ([10]) *An N-linear connection D is called compatible to the fundamental tensor g^{ij} of the Hamiltonian space of order k, $H^{(k)n} = (M, H)$, or it is metrical if g^{ij} is covariant constant (or absolute parallel) with respect to D, i.e.*

$$g^{ij}{}_{|h} = 0,\ g_{ij} \overset{(\alpha)}{|}_h = 0,\ g^{ij}|^h = 0, (\alpha = 1,...,k-1) \quad (14)$$

The operators of Obata's type are given by

$$\Omega^{ij}_{hk} = \frac{1}{2}(\delta^i_h\delta^j_k - g_{hk}g^{ij}), \Omega^{*ij}_{hk} = \frac{1}{2}(\delta^i_h\delta^j_k + g_{hk}g^{ij})$$
(15)

Proposition 1 The operators of Obata's type are covariant constant with respect to any metrical N-linear connection, D

$$\Omega^{ir}_{sj|h} = 0, \Omega^{ir}_{sj} \overset{(\alpha)}{|}_h = 0, \Omega^{ir}_{sj}|^h = 0,$$
$$\Omega^{*ir}_{sj|h} = 0, \Omega^{*ir}_{sj} \overset{(\alpha)}{|}_h = 0, \Omega^{*ir}_{sj}|^h = 0,$$
where

$_{|h}, \overset{(\alpha)}{|}_h$ *and* $|^h$, denote the $h-, v_\alpha -$ and $w_k -$ covariant derivatives with respect to D, $(\alpha = 1,...,k-1)$.

Let $S_2(T^{*k}M)$ be the set of all symmetric d-tensor fields, of the type (0,2) on $T^{*k}M$, $k \geq 2, k \in N$. As is easily shown, the relations for $a_{ij}, b_{ij} \in S_2(T^{*k}M)$ defined by:

$$(a_{ij} \approx b_{ij}) \Leftrightarrow$$
$$((\exists)\lambda(x, y^{(1)},..., y^{(k-1)}, p) \in F(T^{*k}M),$$
$$a_{ij}(x, y^{(1)},..., y^{(k-1)}, p) = \quad (16)$$
$$= e^{2\lambda(x,y^{(1)},...,y^{(k-1)},p)} b_{ij}(x, y^{(1)},..., y^{(k-1)}, p))$$

is an equivalence relation on $S_2(T^{*k}M)$.

Definition 2 *The equivalent class $\hat{g}$ of $S_2(T^{*k}M)/\approx$ to which the fundamental d-tensor field g_{ij} belongs, is called conformal metrical d-structure on $T^{*k}M$.*

Thus

$$\hat{g} = \{g' \,|\, g'_{ij}(x, y^{(1)},..., y^{(k-1)}, p) =$$
$$= e^{2\lambda(x,y^{(1)},...,y^{(k-1)},p)} g_{ij}(x, y^{(1)},..., y^{(k-1)}, p), \quad (17)$$
$$\lambda(x, y^{(1)},..., y^{(k-1)}, p) \in F(T^{*k}M)\}.$$

Definition 3 *An N-linear connection, D, with local coefficients*

$$D\Gamma(N) = \left(H^i{}_{jh}, \underset{(\alpha)}{C}{}^i{}_{jh}, C_i{}^{jh}\right), (\alpha = 1,...,k-1),$$

is called general conformal metrical N-linear connection with respect to $\hat{g}$ if:

$$g_{ij|h} = K_{ijh}, g_{ij} \overset{(\alpha)}{|}_h = \underset{(\alpha)}{Q}{}_{ijh}, g_{ij}|^h = \dot{Q}_{ij}{}^h, \quad (18)$$

where $_{|h}, \overset{(\alpha)}{|}_h$ *and* $|^h$, denote the $h-, v_\alpha -$ and $w_k -$ covariant derivatives with respect to D and $K_{ijh}, \underset{(\alpha)}{Q}{}_{ijh}, \dot{Q}_{ij}{}^h$ are arbitrary tensor fields on $T^{*k}M$ of the types (0,3), (0,3) and (2,1) respectively, with the properties:

$$K_{ijh} = K_{jih}, \underset{(\alpha)}{Q}{}_{ijh} = \underset{(\alpha)}{Q}{}_{jih}, \dot{Q}_{ij}{}^h = \dot{Q}_{ji}{}^h, \quad (19)$$
$$(\alpha = 1,...,k-1).$$

Definition 4 *An N-linear connection, D, with local coefficients*

$$D\Gamma(N) = \left(H^i{}_{jh}, \underset{(\alpha)}{C}{}^i{}_{jh}, C_i{}^{jh}\right), (\alpha = 1,...,k-1),$$

for which there exists the 1-form ω,

$$\omega = \omega_i dx^i + \underset{(1)}{\dot{\omega}_i}\,\delta y^{(1)i} + ... + \underset{(k-1)}{\dot{\omega}_i}\,\delta y^{(k-1)i} + \ddot{\omega}^i \delta p_i,$$

such that

$$\begin{cases} g_{ij|h} = 2\omega_h g_{ij}, \; g_{ij} \overset{(\alpha)}{|}_h = \underset{(\alpha)}{\dot{\omega}_h}\, g_{ij}, \\ g_{ij}\,|^h = 2\ddot{\omega}^h g_{ij}, \alpha = 1,...,k-1, \end{cases} \quad (20)$$

where $_{|h}$, $\overset{(\alpha)}{|}_h$ *and* $|^h$, denote the $h-, v_\alpha -$ and $w_k -$ covariant derivatives with respect to D, $(\alpha = 1,...,k-1)$ is called conformal metrical N-linear connection, with respect to the conformal metrical d-structure $\hat{g}$, corresponding to the 1-form ω and it is denoted by: $D\Gamma(N,\omega)$.

Proposition 2 *If* $D\Gamma(N,\omega)$ $= \left(H^i{}_{jh}, \underset{(\alpha)}{C}{}^i{}_{jh}, C_i{}^{jh} \right) (\alpha = 1,...,k-1)$ *are the local coefficients of a conformal metrical N-linear connection in* $T^{*k}M$*, with respect to the conformal metrical structure* $\hat{g}$*, corresponding to the 1-form* ω*, then*

$$g^{ij}{}_{|h} = -2\omega_h g^{ij}, \; g^{ij} \overset{(\alpha)}{|}_h = -2\underset{(\alpha)}{\dot{\omega}_h}\, g^{ij},$$
$$g^{ij}\,|^h = -2\ddot{\omega}^h g^{ij}, \alpha = 1,...,k-1.$$

For any representative $g' \in \hat{g}$ we have

Theorem 3 *For* $g'_{ij} = e^{2\lambda} g_{ij}$*, a conformal metrical N-linear connection with respect to the conformal metrical structure* $\hat{g}$*, corresponding to the 1-form* ω *in* $T^{*k}M$, $D\Gamma(N,\omega)$*, satisfies*

$$g'_{ij|h} = 2\omega'_h g'_{ij}, \; g'_{ij} \overset{(\alpha)}{|}_h = 2\underset{(\alpha)}{\dot{\omega}'_h}\, g'_{ij},$$
$$g'_{ij}\,|^h = 2\ddot{\omega}'^h g'_{ij}, \alpha = 1,...,k-1,$$

where $\omega' = \omega + d\lambda$.

Since in Theorem 3 $\omega' = 0$ is equivalent to $\omega = d(-\lambda)$, we have

Theorem 4 *A conformal metrical N-linear connection with respect to* $\hat{g}$*, corresponding to the 1-form* ω *in* $T^{*k}M$, $D\Gamma(N,\omega)$*, is metrical with respect to* $g' \in \hat{g}$*, i.e.* $g'_{ij|k} = g'_{ij} \overset{(\alpha)}{|}_k = g'_{ij}\,|^k = 0$ *if and only if* ω *is exact.*

We shall determine the set of all general conformal metrical N-linear connections, with respect to $\hat{g}$, corresponding to the same nonlinear connection N.

Let

$$\overset{0}{D}\Gamma(N) = \left(\overset{0}{H}{}^i{}_{jh}, \underset{(\alpha)}{\overset{0}{C}}{}^i{}_{jh}, \overset{0}{C}{}_i{}^{jh} \right), \quad (\alpha = 1,...,k-1)$$

be the local coefficients of a fixed N- linear connection $\overset{0}{D}$, where

$(\underset{(\alpha)}{N}{}^j{}_i(x, y^{(1)},..., y^{(k-1)}, p), N_{ij}(x, y^{(1)},..., y^{(k-1)}, p)),$

$(\alpha = 1,...,k-1), (i, j = 1,2,...,n)$

are the local coefficients of the nonlinear connection N. Then any N-linear connection, D, with the local coefficients $D\Gamma(N) = \left(H^i{}_{jh}, \underset{(\alpha)}{C}{}^i{}_{jh}, C_i{}^{jh} \right), (\alpha = 1,...,k-1)$, can be expressed in the form (11).

Using the relations (20), (11), (6) and the Theorem 1 given by R.Miron in ([4]) for the case of Finsler connections we obtain

Theorem 5 *Let* $\overset{0}{D}$ *be a given* N *-linear connection, with local coefficients* $\overset{0}{D}\Gamma(N)$ $= \left(\overset{0}{H}{}^i{}_{jh}, \underset{(\alpha)}{\overset{0}{C}}{}^i{}_{jh}, \overset{0}{C}{}_i{}^{jh} \right)$ $(\alpha = 1,...,k-1)$. The set of all general conformal metrical N-linear connections, with respect to $\hat{g}$, corresponding to the same nonlinear connection N with local coefficients

$D\Gamma(N) = \left(H^i{}_{jh}, \underset{(\alpha)}{C}{}^i{}_{jh}, C_i{}^{jh} \right), (\alpha = 1,...,k-1)$ is given by

$$\begin{cases} H^i{}_{jh} = \overset{0}{H}{}^i{}_{jh} + \dfrac{1}{2} g^{im} (g_{mj\overset{0}{|}h} - K_{mjh}) + \Omega^{ir}_{sj} X^s{}_{rh}, \\ \underset{(\alpha)}{C}{}^i{}_{jh} = \underset{(\alpha)}{\overset{0}{C}}{}^i{}_{jh} + \dfrac{1}{2} g^{im} (g_{mj} \overset{\overset{0}{(\alpha)}}{|}_h - \underset{(\alpha)}{Q}{}_{mjh}) + \Omega^{ir}_{sj} \underset{(\alpha)}{Y}{}^s{}_{rh} \\ C_i{}^{jh} = \overset{0}{C}{}_i{}^{jh} + \dfrac{1}{2} g^{mj} (g_{mj} \overset{0}{|}{}^h - \dot{Q}_{mi}{}^h) + \Omega^{jr}_{si} Z_r{}^{sh}, \end{cases} \quad (21)$$

$(\alpha = 1,...,k-1)$

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where $\overset{0}{_{|h}}$, $\overset{(\alpha)}{\overset{0}{|}}{}_h$, and $\overset{0}{|}{}^h$ denote the $h-, v_\alpha-$ and w_k- covariant derivatives with respect to $\overset{0}{D}$, $X^i{}_{jh}, \underset{(\alpha)}{Y}{}^i{}_{jh}, Z_i^{jh}$ are arbitrary d-tensor fields and $K_{ijh}, \underset{(\alpha)}{Q}{}_{ijh}, \dot{Q}_{ij}{}^h$ are arbitrary d-tensor fields of the types (0,3), (0,3) and (2,1) respectively, with the properties (19), $(\alpha = 1,...,k-1)$.

Particular cases

1. If we take

$$K_{ijh} = 2\omega_h g_{ij}, \underset{(\alpha)}{Q}{}_{ijh} = 2\underset{(\alpha)}{\dot{\omega}_h} g_{ij}, (\alpha = 1,...,k-1),$$

$$\dot{Q}_{ij}{}^h = 2\ddot{\omega}^h g_{ij}$$

in Theorem 5, we obtain

Theorem 6 *Let $\overset{0}{D}$ be a given N -linear connection, with local coefficients $\overset{0}{D}\Gamma(N) = \left(\overset{0}{H}{}^i{}_{jh}, \underset{(\alpha)}{\overset{0}{C}}{}^i{}_{jh}, \overset{0}{C}{}_i^{jh}\right)$ $(\alpha = 1,...,k-1)$. The set of all conformal metrical N-linear connections with respect to $\hat{g}$, corresponding to the 1-form ω, with local coefficients $D\Gamma(N,\omega) = \left(H^i{}_{jh}, \underset{(\alpha)}{C}{}^i{}_{jh}, C_i^{jh}\right), (\alpha = 1,...,k-1)$ is given by*

$$\begin{cases} H^i{}_{jh} = \overset{0}{H}{}^i{}_{jh} + \frac{1}{2} g^{im}(g_{mj\overset{0}{|}h} - 2\omega_h g_{mj}) + \Omega^{ir}_{sj} X^s{}_{rh}, \\ \underset{(\alpha)}{C}{}^i{}_{jh} = \underset{(\alpha)}{\overset{0}{C}}{}^i{}_{jh} + \frac{1}{2} g^{im}(g_{mj} \overset{(\alpha)}{\overset{0}{|}}{}_h - 2\underset{(\alpha)}{\dot{\omega}_h} g_{mj}) + \Omega^{ir}_{sj} \underset{(\alpha)}{Y}{}^s{}_{rh}, \\ C_i^{jh} = \overset{0}{C}{}_i^{jh} + \frac{1}{2} g^{mj}(g_{mi} \overset{0}{|}{}^h - 2\ddot{\omega}^h g_{mi}) + \Omega^{jr}_{si} Z_r^{sh}, \end{cases} \quad (22)$$

$(\alpha = 1,...,k-1), (i,j,h = 1,2,...,n)$,

where $\overset{0}{_{|h}}$, $\overset{(\alpha)}{\overset{0}{|}}{}_h$, and $\overset{0}{|}{}^h$ denote the $h-, v_\alpha-$ and w_k- covariant derivatives with respect to $\overset{0}{D}$, $X^i{}_{jh}, \underset{(\alpha)}{Y}{}^i{}_{jh}, Z_i^{jh}$ are arbitrary d-tensor fields, $(\alpha = 1,...,k-1)$, $\omega = \omega_i dx^i + \underset{(1)}{\dot{\omega}_i} \delta y^{(1)i} + ... + \underset{(k-1)}{\dot{\omega}_i} \delta y^{(k-1)i} + \ddot{\omega}^i \delta p_i$, is an arbitrary 1-form and Ω is the operator of Obata's type given by (15).

2. If $X^i{}_{jh} = \underset{(\alpha)}{Y}{}^i{}_{jh} = Z_i^{jh} = 0$, in Theorem 5 we have an example of general conformal metrical with respect to $\hat{g}$:

Theorem 7 *Let $\overset{0}{D}$ be a given N -linear connection, with local coefficients $\overset{0}{D}\Gamma(N) = \left(\overset{0}{H}{}^i{}_{jh}, \underset{(\alpha)}{\overset{0}{C}}{}^i{}_{jh}, \overset{0}{C}{}_i^{jh}\right)$* $(\alpha = 1,...,k-1)$. Then the following N-linear conection K, with local coefficients

$$K\Gamma(N) = \left(H^i{}_{jh}, \underset{(\alpha)}{C}{}^i{}_{jh}, C_i^{jh}\right), (\alpha = 1,...,k-1),$$

given by (23) is general conformal metrical with respect to $\hat{g}$.

$$\begin{cases} H^i{}_{jh} = \overset{0}{H}{}^i{}_{jh} + \frac{1}{2} g^{im} (g_{mj \overset{0}{|} h} - K_{mjh}), \\ \underset{(\alpha)}{C}{}^i{}_{jh} = \underset{(\alpha)}{\overset{0}{C}}{}^i{}_{jh} + \frac{1}{2} g^{im} (g_{mj} \overset{(\alpha)}{\overset{0}{|}}{}_h - \underset{(\alpha)}{Q}{}_{mjh}), \\ C_i{}^{jh} = \overset{0}{C}{}_i{}^{jh} + \frac{1}{2} g^{mj} (g_{mi} \overset{0}{|}{}^h - \dot{Q}_{mi}{}^h), \end{cases} \quad (23)$$

$(\alpha = 1,...,k-1)$,

where ${}_{\overset{0}{|}h}$, $\overset{(\alpha)}{\overset{0}{|}}{}_h$, and $\overset{0}{|}{}^h$ denote the $h-, v_\alpha -$ and $w_k -$ covariant derivatives with respect to $\overset{0}{D}$, and $K_{ijh}, \underset{(\alpha)}{Q}{}_{ijh}, \dot{Q}_{ij}{}^h$ are arbitrary d-tensor fields of the types (0,3), (0,3) and (2,1) respectively, with the properties (19), $(\alpha = 1,...,k-1)$.

3. If we take a general conformal metrical N-linear connection with respect to $\hat{g}$ as $\overset{0}{D}$, in Theorem 5 we have

Theorem 8 *Let* $\overset{0}{D}$ *be on* $T^{*k}M$ *a fixed general conformal metrical N-linear connection with respect to* $\hat{g}$*, with the local coefficients*

$$\overset{0}{D}\Gamma(N) = \left(\overset{0}{H}{}^i{}_{jh}, \underset{(\alpha)}{\overset{0}{C}}{}^i{}_{jh}, \overset{0}{C}{}_i{}^{jh} \right) \quad (\alpha = 1,..., \quad k-1).$$

The set of all general conformal metrical N-linear connections, with respect to $\hat{g}$, with local coefficients

$D\Gamma(N) = \left(H^i{}_{jh}, \underset{(\alpha)}{C}{}^i{}_{jh}, C_i{}^{jh} \right), (\alpha = 1,...,k-1)$ is given by

$$\begin{cases} H^i{}_{jh} = \overset{0}{H}{}^i{}_{jh} + \Omega^{ir}_{sj} X^s{}_{rh}, \\ \underset{(\alpha)}{C}{}^i{}_{jh} = \underset{(\alpha)}{\overset{0}{C}}{}^i{}_{jh} + \Omega^{ir}_{sj} \underset{(\alpha)}{Y}{}^s{}_{rh}, (\alpha = 1,...,k-1), \\ C_i{}^{jh} = \overset{0}{C}{}_i{}^{jh} + \Omega^{jr}_{si} Z_r{}^{sh}, \end{cases} \quad (24)$$

where $X^i{}_{jh}, \underset{(\alpha)}{Y}{}^i{}_{jh}, Z_i{}^{jh}$ are arbitrary d-tensor fields, $(\alpha = 1,...,k-1)$.

4. If $K_{ijh} = \underset{(\alpha)}{Q}{}_{ijh} = \dot{Q}_{ij}{}^h = 0, (\alpha = 1,...,k-1)$ in Theorem 5 we obtain the set of all metrical N-linear connection in the case when the nonlinear connection is fixed, result given in ([10]).

Theorem 9 *The mappings determined by* (24), $D\Gamma(N) \to \bar{D}\Gamma(N)$ *together with the composition of these mappings is an abelian group.*

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