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MATHEMATICAL MODEL FOR A JET ENGINE WITH COOLING FLUID INJECTION INTO ITS COMBUSTOR

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Abstract: *The paper deals with an aircraft jet engine with cooling fluid injection into the rear part of its combustor, meant to temporarily increase its thrust, treated as controlled object. The author has established system's motion equations, consecutive to the new gas-dynamic and fluid mechanics conditions. Using the equation system, the author has obtained engine's new structure matrix description, as well as its transfer functions. A study concerning its time behavior was performed (about its speed, combustor temperature and thrust) and some conclusions were presented, comparing to other cooling fluid injection methods results. The paper is useful for students and researchers in their jet engine automation studies and may be improved by considering the flight regime influence.*

Keywords: *engine, cooling fluid, injection, combustor, thrust augmentation, flow rate.*

1. INTRODUCTION

One of the aircraft jet engines' thrust increasing methods consists of fluid injection in the rear part of its combustor, in the mixing area. The phenomena are described in [3,4,5] and thermo-dynamically explained and grounded in [5].

The aircraft engine may be overboosted using afterburning systems or alternative thrust augmentation methods. The afterburning is the most efficient thrust augmentation method, but in the same time, it is the most expensive-one, because of its fuel consumption increasing, as well as because of its mandatory constructive modifications and automatic control schemes implementation. Meanwhile, the afterburning isn't an appropriate thrust augmentation system for turboprop engines, nor for twin-jet turbofan engines in their outer jet. Especially for turboprops, the lack of air and the presence of the propeller make impossible the afterburning adapting as well as a high flight speed achieving.

For these propulsion systems, alternative thrust augmentation methods are: a) fluid injection into the engine's compressor [5,10]; b) fluid injection into the engine's combustor, in its rear part (as figure 1 shows). Both these methods are meant to increase the exhaust nozzle enthalpy fall, by reducing the turbine enthalpy fall, consecutive to a smaller compressor mechanical work necessity.

The first method was described in [5] and engine's new mathematical model was determined and studied in [10].

The second method is the subject of this paper, which intends to establish the new mathematical model of the engine, as well as to study engine's time behavior (by studying

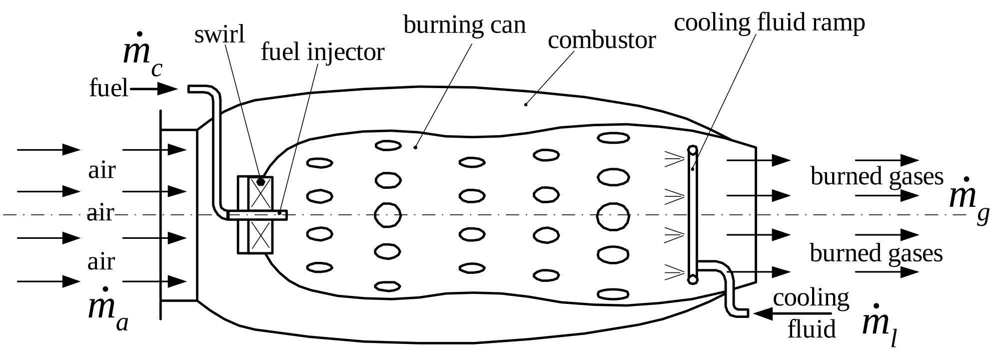

Figure 1. Aircraft jet engine's combustor with cooling fluid injection facility

the step response of the engine concerning its speed, temperature and thrust).

In both of the above-mentioned cases, it results an important thrust augmentation, caused by both the mass airflow increasing and the exhaust nozzle burned gases' speed increasing, until 60% for the first method and until 25% for the second method, corresponding to a cooling fluid flow rate fraction of 5% of the engines air flow rate. Figure 2 shows thrust and specific fuel consumption growing percentages with respect to the injected fuel fraction. Fuel consumption is bigger than in the case studied in [10] (that means in the case of cooling fluid injection into the engine's compressor).

One can observe that engine's performances are a little smaller than in the first case, but this method offers some advantages, such as constructive simplicity, the elimination of the icing possibilities, as well as the elimination of the blades corrosion. A major disadvantage is that an uncontrolled fluid injection could obstruct an appropriate burning deployment and it could even extinguish the combustor's flame; that is the reason why the

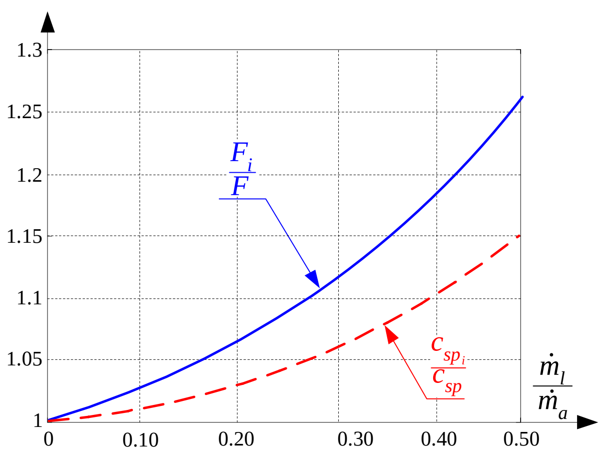

Figure 2. Engine's performances with respect to the injected fluid flow rate

injection is done at the rear part of the burner can, near its wall, in order to assure a supplementary cooling of the burner's wall and, meanwhile, to facilitate the mixing of the burned gases with the vaporized cooling fluid.

In most of the practical situations the used cooling fluid is the water, which means a neutral fluid, the injection of a combustible fluid being unnecessary, even prohibited.

In some situations an air flow rate by-passing is necessary (see figure 3), when the compressor's air flow rate exceeds the necessary, in order to prevent an unstable engine operation (stall).

2. THERMODYNAMIC EFFECTS OF THE FLUID INJECTION

2.1 Flow rate balance. The fluid injection brings into the combustor (and into the general flow) an extra flow rate $\dot{m}_l$.

Most of the nowadays operational jet engines have critical flow in their turbines [5], so the flow parameter $\frac{\dot{m}_g\sqrt{T_3^*}}{p_3^*}$ remains constant even if one uses the fluid injection, where $\dot{m}_g$ is the burned gases flow rate, T_3^* – gas temperature before the turbine, p_3^* – gas pressure before the turbine, proportional to the air pressure after the compressor $\left(p_3^* = \sigma_{CA}^* p_2^*\right)$.

The greater the injected fluid flow rate $\dot{m}_l$, the less the compressor air flow $\dot{m}_a$ rate must be and it leads to an important pressure increasing (both p_2^* and p_3^*); meanwhile, the operating regime becomes closer to the stall limit, which is an undesirable phenomenon. Consequently, the burned gases flow rate must keep its value and becomes fuel, which is a supplementary propulsion system, offering its own thrust.

Obviously, as long as the extracted $\dot{m}_{ap}$ air flow rate value depends on the cooling fluid flow rate $\dot{m}_l$ value, the presence of an appropriate fuel control system for this new combustor is compulsory, in order to correlate the fuel flow rate injection to the air flow rate.

Meanwhile, the main fuel flow rate control

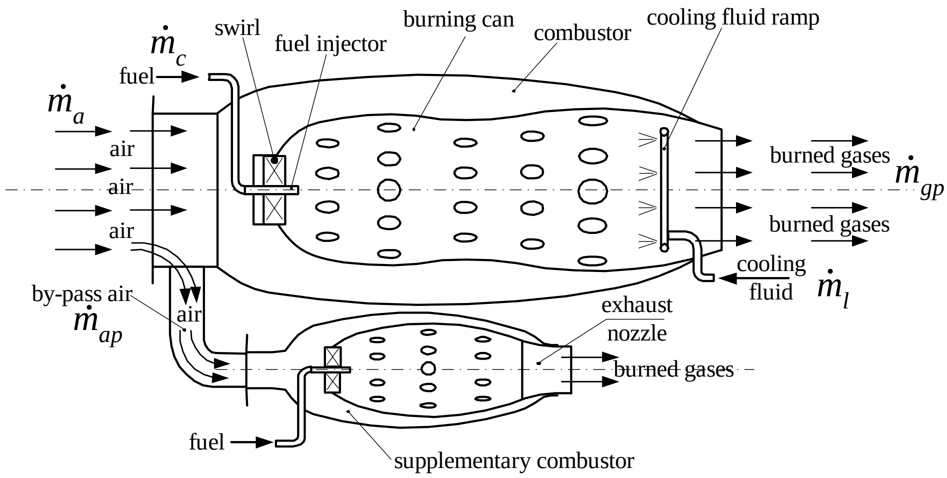

Figure 3. Jet engine's combustor with cooling fluid injection, air by-pass and supplementary combustor

$$\dot{m}_g = \dot{m}_{ai} + \dot{m}_l + \dot{m}_c\,, \tag{1}$$

where $\dot{m}_{ai}$ is the new air flow rate value, smaller than the initial value $\dot{m}_a$.

In order to avoid unstable regimes, even when the cooling fluid injection is operational, a combined thrust augmentation method can be implemented. This method is meant to keep constant the compressor's air flow rate and, meanwhile, to extract (by-pass) an air flow rate $\dot{m}_{ap}$ before the engine's combustor, in order to keep the flow rate balance

$$\dot{m}_{gp} = \dot{m}_a - \dot{m}_{ap} + \dot{m}_l + \dot{m}_c \equiv \dot{m}_g\,. \tag{2}$$

The extracted air flow rate is not a loss for the system; it can be used into another combustor (an external, independent, supplementary combustor), provided with its own exhaust nozzle and separately supplied with

system must be reset for the new conditions.

The flow rate balance must be kept in both of situations, without cooling fluid injection (basic engine) or with cooling fluid injection.

For the basic engine

$$\dot{m}_g = \mu A_{3_{cr}}^{/} \frac{p_3^*}{\sqrt{T_3^*}} \sqrt{\frac{\chi_g}{R_g}\left(\frac{2}{\chi_g+1}\right)^{\frac{\chi_g+1}{\chi_g-1}}}\,, \tag{3}$$

while for the engine with fluid injection

$$\dot{m}_{gp} = \mu A_{3_{cr}}^{/} \frac{p_3^*}{\sqrt{T_3^*}} \sqrt{\frac{\chi_{gp}}{R_{gp}}\left(\frac{2}{\chi_{gp}+1}\right)^{\frac{\chi_{gp}+1}{\chi_{gp}-1}}}\,, \tag{4}$$

where μ is the flow rate co-efficient, $A_{3_{cr}}^{/}$ – turbine's stator critical area, χ_g, χ_{gp} – adiabatic exponents of the burned gases, R_g, R_{gp} –

gas constants of the burned gases (for both situations).

Considering the formal annotation for flow rate fractions (with respect to the $\dot{m}_a$ air flow rate) as $\xi_x = \frac{\dot{m}_x}{\dot{m}_a}$, from Eqs. (2), (3) and (4) one obtains

$$\frac{(1-\xi_{ap})(1+\xi_l+\xi_c)}{(1-\xi_{ci})} = \sqrt{\frac{R_g}{R_{gp}} \frac{\left(\frac{2}{\chi_{gp}+1}\right)^{\frac{\chi_{gp}+1}{\chi_{gp}-1}}}{\left(\frac{2}{\chi_g+1}\right)^{\frac{\chi_g+1}{\chi_g-1}}}}, \tag{5}$$

where ξ_c, ξ_{ci} – fuel flow rate fractions for the basic engine, respectively for the engine with cooling fluid injection (determined as $\xi_c = \frac{1}{\lambda_{CA} \min L}$), $\min L$ – stoechiometric minimum air value for 1 kg of fuel, λ_{CA} – air excess co-efficient in engine's combustor, ξ_{ap} – extracted air flow fraction and ξ_l – injected fluid fraction.

Assuming that burned gases properties are nearly the same for both of situations, the right member in Eq. (5) becomes equal to 1, so

$$\xi_{ci} = \xi_c - \xi_l + \xi_{ap}(1+\xi_l). \tag{6}$$

2.2 Main combustor's energy balance. Energy balance equation for the combustor must be written in both situations (without and with cooling fluid injection).

For the basic engine the equation is

$$\dot{m}_a i_2^* + \dot{m}_c(\zeta_{CA} P_{ci} + i_c) = \dot{m}_g i_3^* = \dot{m}_a (1+\xi_c) i_3^*, \tag{7}$$

while for the engine with fluid injection

$$\dot{m}_{ai} i_2^* + \dot{m}_{ci}(\zeta_{CA} P_{ci} + i_c) + \dot{m}_l i_l = \dot{m}_{gp} i_{3i}^*. \tag{8}$$

Given the formulas (1), (2) and (6), after dividing Eqs. (7) and (8) by $\dot{m}_a$, one obtains

$$i_2^* + \xi_c(\zeta_{CA} P_{ci} + i_c) = (1+\xi_c) i_3^*, \tag{9}$$

$$i_2^* + \xi_{ci}(\zeta_{CA} P_{ci} + i_c) + \xi_l i_l = (1 - \xi_{ap} + \xi_l + \xi_c) i_{3i}^* \tag{10}$$

where i_c, i_l are fuel's and injected liquid's specific enthalpy, i_2^*, i_3^*, i_{3i}^* – air/burned gases specific enthalpy, P_{ci} – fuel's chemical energy, ζ_{CA} – combustor's burning perfection co-efficient.

Assuming that burned gases enthalpy must remain constant and, meanwhile, assuming that fuel's enthalpy is very small, negligible compared to fuel's chemical energy (P_{ci}) and that burning perfection co-efficient (ζ_{CA}) remains constant (with or without cooling fluid injection), from the above-determined equation one can express the fluid injection fraction as

$$\xi_l = \frac{1}{i_3^* - i_l}\left[\left(1 + \frac{1}{\lambda_{CA} \min L}\right) i_{3i}^* - i_2^* - \right.$$
$$\left. - \frac{\zeta_{CA} P_{ci} + i_l}{\lambda_{CA} \min L}\right]. \tag{11}$$

Eqs. (6) and (11) may determine the extracted air fraction, with respect to the injected cooling fluid fraction.

3. ENGINE'S NEW MOTION EQUATIONS

Engine's mathematical model consists of:

- engine's spool motion equation;
- compressor's and turbine's characteristics;
- combustor's energy equation;
- air/gases flow rate's equation.

These equations are studied in [8] for a basic engine; a matrix description is also given

$$[A] \times (u) = (b), \tag{12}$$

where $[A]$ is engine's matrix, (u) – output parameters vector and (b) – input parameters vector:

$$A = \begin{bmatrix} T_1 s + \rho_1 & -k_{1T3} & 0 & -k_{1p2} & k_{1p4} \\ k_{2n} & -k_{2T3} & 0 & k_{2p2} & 0 \\ 0 & -1 & 1 & -k_{3p2} & -k_{3p4} \\ 0 & k_{4T3} & k_{4T4} & k_{4p2} & k_{4p4} \\ k_{5n} & k_{5T3} & 0 & k_{5p2} & 0 \end{bmatrix} \tag{13}$$

$$u^T = \begin{pmatrix} \overline{n} & \overline{T_3^*} & \overline{T_4^*} & \overline{p_2^*} & \overline{p_4^*} \end{pmatrix}, \tag{14}$$

$$b^T = \begin{pmatrix} 0 & 0 & 0 & 0 & k_{5Qc}\overline{Q}_c \end{pmatrix}. \tag{15}$$

The involved co-efficient are used with their expressions, described in [8].

Based on previous chapter thermodynamic considerations, one can affirm that the fluid injection has influence on the air/gases flow rate balance along the engine, as well as on the energy balance in the engine's combustor. Consequently, one has to modify the equations involving the temperature behind the engine's combustor, as well as the equation of flow rate's continuity.

One has to neglect the new propulsion system (supplementary combustor), which operates together with the basic engine, because it has no influence above the mathematical model; it may only be considered as a thrust augmentation factor and it must be included, if possible, only in the global thrust calculation.

3.1 Flow rate equation. The exhaust gases flow rate is given by Eq. (10), where the fuel flow rate is the smallest and can be neglected

$$\dot{m}_{g_i}\left(p_3^*, T_3^*\right) = \dot{m}_{a_i}\left(p_2^*, n\right) + \dot{m}_l . \qquad (16)$$

The above equation should be linearised, using the finite differences method, in order to be used into the mathematical model

$$\left(\frac{\partial \dot{m}_{g_i}}{\partial p_3^*}\right)_0 \Delta p_3^* + \left(\frac{\partial \dot{m}_{g_i}}{\partial T_3^*}\right)_0 \Delta T_3^* = \left(\frac{\partial \dot{m}_{a_i}}{\partial p_2^*}\right)_0 \Delta p_2^* + \\ + \left(\frac{\partial \dot{m}_{a_i}}{\partial n}\right)_0 \Delta n + \Delta \dot{m}_l . \qquad (17)$$

Assuming that $p_3^* = \sigma_{CA}^* p_2^*, \sigma_{CA}^* = \text{const.}$, the above equation becomes

$$\left[\left(\frac{\partial \dot{m}_{g_i}}{\partial p_2^*}\right)_0 \sigma_{CA}^* - \left(\frac{\partial \dot{m}_{a_i}}{\partial p_2^*}\right)_0\right] \Delta p_2^* + \left(\frac{\partial \dot{m}_{g_i}}{\partial T_3^*}\right)_0 \Delta T_3^* - \\ - \left(\frac{\partial \dot{m}_{a_i}}{\partial n}\right)_0 \Delta n = \Delta \dot{m}_l , \qquad (18)$$

or

$$\frac{p_{2_0}^*}{\left(\dot{m}_{a_i}\right)_0}\left[\left(\frac{\partial \dot{m}_{g_i}}{\partial p_2^*}\right)_0 \sigma_{CA}^* - \left(\frac{\partial \dot{m}_{a_i}}{\partial p_2^*}\right)_0\right] \frac{\Delta p_2^*}{p_{2_0}^*} + \\ \frac{T_{3_0}^*}{\left(\dot{m}_{a_i}\right)_0}\left(\frac{\partial \dot{m}_{g_i}}{\partial T_3^*}\right)_0 \frac{\Delta T_3^*}{T_{3_0}^*} - \frac{n_0}{\left(\dot{m}_{a_i}\right)_0}\left(\frac{\partial \dot{m}_{a_i}}{\partial n}\right)_0 \frac{\Delta n}{n_0} = \\ = \frac{\left(\dot{m}_l\right)_0}{\left(\dot{m}_{a_i}\right)_0} \frac{\Delta \dot{m}_l}{\left(\dot{m}_l\right)_0} = \xi_l \frac{\Delta \dot{m}_l}{\left(\dot{m}_l\right)_0} . \qquad (19)$$

Assuming the formal annotation $\overline{X} = \dfrac{\Delta X}{X_0}$ the above equation becomes

$$k_{2n}^{\prime} \overline{n} - k_{2T3}^{\prime} \overline{T_3^*} + k_{2p2}^{\prime} \overline{p_2^*} = \xi_l \overline{\dot{m}_l} ; \qquad (20)$$

one can observe that the second line in matrix $[A]$ should be replaced by the new values of the co-efficient. Meanwhile, one has to complete the second element in the input vector (b) with the term in the right member of Eq. (20).

3.2 Combustor's energy equation. The fifth equation in the mathematical model is based on the combustor's energy equation, which may have different forms, according to the injected fluid's nature:

$$\dot{m}_{g_i} c_{p_g} T_3^* - \dot{m}_{a_i} c_{p_a} T_2^* = \dot{m}_c \zeta_{CA} P_c + \dot{m}_l \zeta_{CA} P_l , \qquad (21)$$

where c_{p_g}, c_{p_a} – specific isobar heat of the burned gases and air (assumed as equal), ζ_{CA} – burning process' perfection co-efficient, P_c, P_l – chemical energy of the fuel, respectively of the injected fluid.

Meanwhile, the term T_2^* should be expressed with respect to p_2^*

$$\Delta T_2^* = \left(\frac{\partial T_2^*}{\partial \pi_c^*}\right)_0 \Delta \pi_c^* = \left(\frac{\partial T_2^*}{\partial \pi_c^*}\right)_0 \left(\frac{\partial \pi_c^*}{\partial p_2^*}\right)_0 \Delta p_2^* , \qquad (22)$$

or as

$$\overline{T_2^*} = \frac{p_{2_0}^*}{T_{2_0}^*}\left(\frac{\partial T_2^*}{\partial \pi_c^*}\right)_0 \left(\frac{\partial \pi_c^*}{\partial p_2^*}\right)_0 \overline{p_2^*} . \qquad (23)$$

The cooling fluid, which is injected into the rear part of the combustor is a neutral-one and doesn't participate at the burning reaction

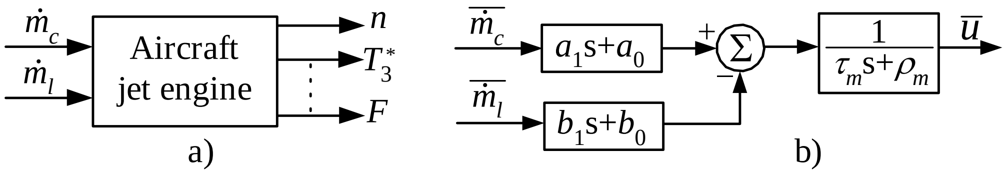

Figure 4. Aircraft jet engine as controlled system (object)

(as flammable substance). Consequently, Eq. (21) becomes

$$\dot m_{g_i} c_p T_3^* - \dot m_{a_i} c_p T_2^* = \dot m_c \zeta_{CA} P_c . \qquad (24)$$

Considering Eqs. (16), (17), (18) and (23), one obtains from (24)

$$\frac{c_p\left(T^*_{3_0}-T^*_{2_0}\right)n_0}{\dot m_{c_0}\zeta_{CA}P_c}\left(\frac{\partial \dot m_a}{\partial n}\right)_0 \overline{n} + \frac{c_p\left(\dot m_{a_0}-\dot m_{l_0}\right)T^*_{3_0}}{\dot m_{c_0}\zeta_{CA}P_c}\overline{T_3^*} + \left[\frac{c_p\left(T^*_{3_0}-T^*_{2_0}\right)p^*_{2_0}}{\dot m_{c_0}\zeta_{CA}P_c}\left(\frac{\partial \dot m_a}{\partial p_2^*}\right)_0 - \frac{c_p\left(\dot m_{a_0}-\dot m_{l_0}\right)}{\dot m_{c_0}\zeta_{CA}P_{ci}} \times p^*_{2_0}\left(\frac{\partial T_2^*}{\partial \pi_c^*}\right)_0\left(\frac{\partial \pi_c^*}{\partial p_2^*}\right)_0\right]\overline{p_2^*} = \overline{\dot m_c} - \frac{\dot m_{l_0}}{\dot m_{c_0}\zeta_{CA}P_{ci}}\overline{\dot m_l} \qquad (25)$$

One can observe that the co-efficient $\frac{\dot m_{l_0}}{\dot m_{c_0}\zeta_{CA}P_{ci}}$ of the injected fluid flow rate parameter $\overline{\dot m_l}$ has a very small value comparing to 1, the value of the co-efficient of $\overline{\dot m_c}$, so it may be neglected. Consequently, the above-determined equation may be re-written as

$$k_{5n}\overline{n} + k'_{5T3}\overline{T_3^*} + k'_{5p2}\overline{p_2^*} = \overline{\dot m_c} \qquad (26)$$

and the last line in matrix [A] should be appropriate restored.

The [A]-matrix, as well as the (b)-vector should be modified in appropriate modes, with respect to the injected fluid nature.

4. SYSTEM'S QUALITY

Jet engine's behavior, as controlled object (system), should be studied for the new conditions. System's quality consists of engine's step response (its time behavior for step input or inputs).

An aircraft engine with combustor fluid injection can be represented, as controlled object, by a system with two inputs (fuel flow rate and injected fluid flow rate) and more outputs (engine speed, combustor temperature, thrust etc), as figure 4.a shows.

Following the algorithm described in [6,7,8], each one of the outputs u can be expressed with respect to the above-mentionned inputs, as formally shown in figure 4.b.

As main outputs the next three were considered: a) engine's speed non-dimensional parameter $\overline{n}$; b) engine's combustor temperature parameter $\overline{T_3^*}$; c) engine's thrust parameter $\overline{F}$.

One has chosen, for a quantitative study, a VK-1A-type jet engine, with constant area exhaust nozzle, having in mind only the engine as possible controlled object, without its control systems (without the speed controller and the temperature limiter). Its flight regime is conventionally chosen as stationary (airspeed V=0) at sea level (altitude H=0).

Output parameters' expressions for the VK-1A basic engine are

$$\overline{n}(s) = \frac{1.2606}{2.0859s + 5.1015}\overline{\dot m_c}, \qquad (30)$$

$$\overline{T_3^*}(s) = \frac{1.3799\,s + 2.3888}{2.0859s + 5.1015}\overline{\dot m_c}, \qquad (31)$$

$$\overline{F}(s) = \frac{1.3762\,s + 4.762}{2.0859s + 5.1015}\overline{\dot m_c}, \qquad (32)$$

depicted with dashed lines for step responses in figures 5, 6 and 7.

Figure 5 shows the engine's speed parameters step response, while figure 6 shows the same response of the combustor temperature parameter and figure 7 contains engine's thrust behavior for the same conditions.

The case of the injection of a neutral cooling fluid (water) into the rear part of the engine's combustor brings next mathematical model modifications

$$\overline{n}(s) = \frac{1}{2.3761s + 4.817}\left(1.411\overline{\dot m_c} - 0.167\overline{\dot m_l}\right), \qquad (33)$$

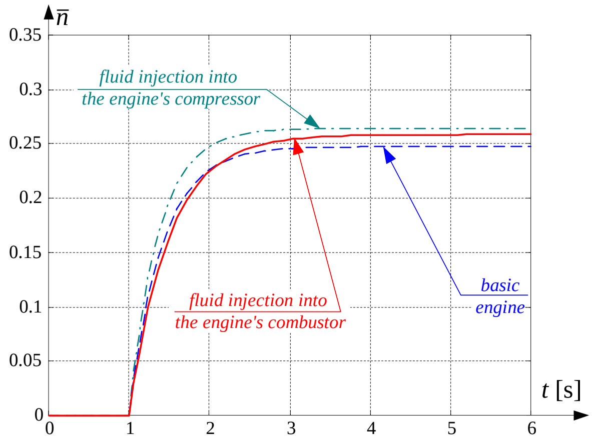

Figure 5. Engine's speed parameter step response

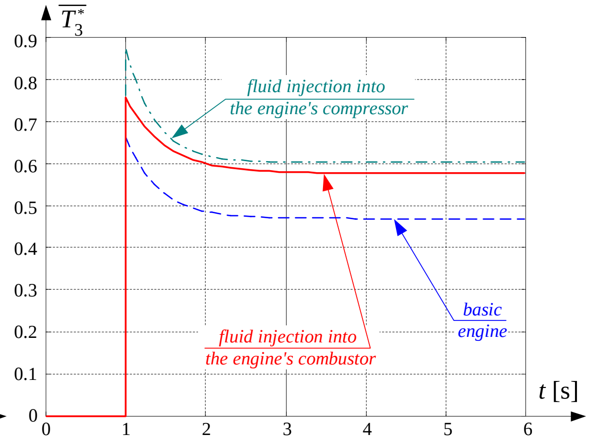

Figure 6. Engine's combustor temperature parameter step response

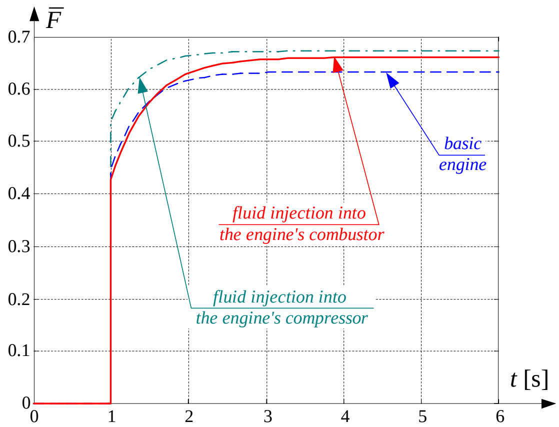

Figure 7. Engine's thrust parameter step response

$$\overline{T_3^*}(s) = \frac{1}{2.3761s + 4.817}\left[(1.8732s + 2.847)\overline{\dot{m}_c} - \right.$$
$$\left. -(0.0823s + 0.0764)\overline{\dot{m}_1}\right] \tag{34}$$

$$\bar{F}(s) = \frac{1}{2.3761s + 4.817}\left[(1.583s + 5.167)\overline{\dot{m}_c} - \right.$$
$$\left. -(0.0834s + 0.4725)\overline{\dot{m}_1}\right] \tag{35}$$

In order to realize a comparison between the performances of the fluid injection thrust augmentation methods, the results of the compressor fluid injection method in [10] were taken and inserted into the diagrams in figures 5, 7 and 7 (dash-dot lines).

5. CONCLUSIONS

Cooling fluid injection into the jet engine's combustor determines gas-dynamic modifycations and performances improvement.

As the technical literature shows, the described method of thrust augmentation through fluid injection into the combustor is a very effective one, especially for high altitudes flights, thrust increasing being significant (until 25%, as figure 2 shows); meanwhile, the specific fuel consumption has a moderate growing (under 15%), definitely acceptable because of the thrust augmentation advantages.

Gas-dynamic changes have as consequences both jet engine's mathematical model changes, as well as performances improvements.

Mathematical model's equation was modified because of the air/gases flow rate's balance changes, as well as because of the new energy balance of the combustor, when a combustible cooling fluid is involved. In order to keep the engine stable, a small air flow rate is extracted before the combustor and used into a supplementary combustor (with its own exhaust nozzle, which operates as an auxiliary propulsion system).

Whatever the cooling fluid injection method were, engine's speed is less influenced (as figure 5 shows), small $\bar{n}$ – parameter's increasing being observed. Moreover, the injection into the combustor involves less speed modifications than the injection into the compressor, but one has observed, in this case, a small response time growing (about 0.5 s), which means that the engine has become somewhat slower than the basic engine. The injection into the compressor makes the engine a little, but insignificant, faster, from the response time point of view.

From the temperature's parameter behavior point of view (see figure 6), whatever the injection method were, one can observe the same trend as for the basic engine. However, a significant growing of $\overline{T_3^*}$ – parameter's value is observed when the injection is used, as a consequence of the supplementary fuel injection, in order to compensate the air exceeding flow rate, so the temperature's parameter tends to restore the basic engine's behavior.

From the thrust-parameter point of view, whatever the injection method is, thrust presents moderate increase (because of the neutral injected fluids, which doesn't assure supplementary heat input and the thrust augmentation is realized by the air flow and gases speed increase). As figure 7 shows, thrust augmentation is more rapid when the compressor injection is used, but the growing percentage is comparable for both of the used methods.

Engine's time constant is about 15% bigger than for the basic engine; whatever the method were, stabilization time values are kept around (2.5÷3.5) sec, which is acceptable from the practical point of view.

One has performed the study for an engine VK-1A-type, for sea level conditions. This study could be extended for other flight conditions (low altitude and take-off air speed, as well as for high altitudes and cruise air speed), given that the combustor fluid injection thrust augmentation method is useful for airplane's (aircraft) both for taking-off and for medium-high altitude thrust restore.

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