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MATHEMATICAL MODEL FOR A JET ENGINE WITH COOLING FLUID INJECTION INTO ITS COMPRESSOR

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Abstract: *The paper deals with an aircraft jet engine with cooling fluid injection in its compressor, meant to temporarily increase its thrust, treated as controlled object. The author has established system's motion equations, consecutive to the new gas-dynamic and fluid mechanics conditions. Using the equation system, the author has obtained engine's new structure matrix description, as well as its transfer function. A study concerning its time behavior, for two different fluids, was eventually performed (about its speed, combustor temperature and thrust) and some conclusions were presented. The paper is useful for students and researchers in their jet engine automation studies and may be improved by considering the flight regime influence, where possible.*

Keywords: *engine, control, cooling, injection, compressor, thrust augmentation, step response.*

1. INTRODUCTION

One of the aircraft jet engines' thrust increasing methods consists of fluid injection in the front of its compressor. The phenomena are described in [3,4,5] and thermo-dynamically explained and grounded in [5].

From the practical point of view, the aircraft engine may be overboosted using afterburning systems or alternative thrust augmentation methods. The afterburning is the most efficient thrust augmentation method, but in the same time, it is the most expensive-one, because of its fuel consumption increasing, as well as because of its mandatory constructive modifications and automatic control schemes implementation. Meanwhile, the afterburning isn't an appropriate thrust augmentation system for turboprop engines, nor for twin-jet turbofan engines. Especially for turboprops, the lack of air and the presence of the propeller make impossible the afterburning adapting as well as a high flight speed achieving.

For these propulsion systems, alternative thrust augmentation methods are: a) fluid injection into the engine's compressor; b) fluid injection into the engine's combustor. Both these methods are meant to increase the exhaust nozzle enthalpy fall, by reducing the turbine enthalpy fall, consecutive to a smaller compressor mechanical work necessity.

The first method consists of the injection of a special volatile fluid (water, ammonia, methanol, water-methanol mixture etc) into the front of the engine's axial compressor. During the air compression evolution the temperature grows; the injected fluid vaporizes itself and extracts an important part of the resulted heat, which cools the compressed air and grows its mass air flow rate, keeping constant the volume flow rate. The extraction of that heat from the compressed air transforms the adiabatic (isentropic) compression evolution into a polytropic one and determines a mechanical work necessity decreasing.

It results an important thrust augmentation, caused by both the mass airflow increasing and the exhaust nozzle burned gases' speed increasing.

2. THERMODYNAMIC EFFECTS OF THE FLUID INJECTION

2.1 Compression evolution. The heat extraction through the fluid injection transforms the adiabatic compression evolution into a polytropic one, the polytropic exponent being smaller than 1.4 (which is the adiabatic exponent for air, denoted as k). The greater the injected fluid flow rate $\dot{m}_l$ is, the lower the polytropic exponent is, tending to 1 (which corresponds to the ideal isothermic evolution), as figure 1 shows.

The curve in figure 1 was obtained applying the following algorithm [5]. Firstly, one considers the irreversible adiabatic air evolution in the compressor as a polytropic one, which leads to

$$i_1^* \frac{\left(\pi_c^*\right)^{\frac{k-1}{k}}-1}{\eta_c} = i_1^* \frac{k-1}{k} \frac{a}{a-1}\left[\left(\pi_c^*\right)^{\frac{a-1}{a}}-1\right], \quad (1)$$

where i_1^* is the air specific enthalpy in the front of the compressor, π_c^* – compressor's pressure ratio, η_c – compressor's randament, a – polytropic exponent of the equivalent evolution. This non-linear equation's solving gives the equivalent n – value.

The injected fluid may extract the heat

$$Q_i = \dot{m}_l r_f \,, \quad (2)$$

where r_f is fluid's vaporization latent heat, which means, for the air flow rate $\dot{m}_a$, that

$$q_i = \frac{Q_i}{\dot{m}_a} = \frac{\dot{m}_l}{\dot{m}_a} r_f \quad (3)$$

and the specific mechanical work of the compressor diminishes exactly with this value. Consequently, the new polytropic evolution (by a_i exponent) should have the same mechanical work value, so

$$\frac{a}{a-1}\left[\left(\pi_c^*\right)^{\frac{a-1}{a}}-1\right] = \frac{a_i}{a_i-1}\left[\left(\pi_{c_i}^*\right)^{\frac{a_i-1}{a_i}}-1\right], \quad (4)$$

$$i_1^* \frac{a_i}{a_i-1}\left[\left(\pi_{c_i}^*\right)^{\frac{a_i-1}{a_i}}-1\right] = i_1^*\left[\frac{\left(\pi_c^*\right)^{\frac{k-1}{k}}-1}{\eta_c}\right] - q_i \quad (5)$$

where $\pi_{c_i}^*$ is the new compressor's pressure ratio (when the fluid injection is active)

$$\pi_{c_i}^* = \left\{1 + \frac{a_i-1}{a_i}\frac{a}{a-1}\left[\left(\pi_c^*\right)^{\frac{a-1}{a}}-1\right]\right\}^{\frac{a_i}{a_i-1}}. \quad (6)$$

From Eqs. (5) and (6) one obtains the relation between $\dot{m}_l$ and a_i

$$\frac{\left(\pi_c^*\right)^{\frac{k-1}{k}}-1}{\eta_c} = \frac{a_i-1}{a_i}\frac{a}{a-1}\left[\left(\pi_c^*\right)^{\frac{a-1}{a}}-1\right] + \frac{\dot{m}_l}{\dot{m}_a}\frac{r_f}{i_1^*} \quad (7)$$

which is the reason of the graphics in figure 1.

The determined value for a_i, used by the formula (6) leads to the new value of $\pi_{c_i}^*$, so one can obtain the new operation line position on the compressor characteristic.

2.2 Flow rate. Most of the nowadays operational jet engines have critical flow in their turbines [5], so the flow parameter $\frac{\dot{m}_g\sqrt{T_3^*}}{p_3^*}$ remains constant even if one uses the fluid injection, where $\dot{m}_g$ is the burned gas flow rate, T_3^* – gas temperature before the

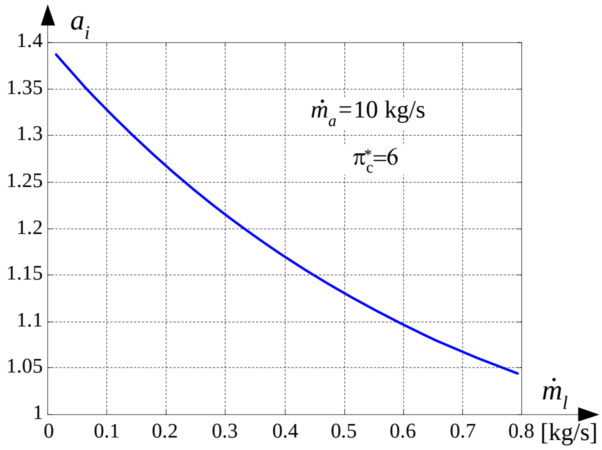

Figure 1. Polytropic exponent with respect to the injected fluid flow rate

turbine, p_3^* – gas pressure before the turbine, proportional to the air pressure after the compressor $\left(p_3^* = \sigma_{CA}^* p_2^*\right)$. Consequently, the air/gases flow rate through the engine must satisfy it, as follows

$$\frac{\dot{m}_g \sqrt{T_3^*}}{p_3^*} = \frac{\dot{m}_a \sqrt{T_3^*}}{\sigma_{CA}^* p_2^*} = \frac{\dot{m}_{a_i} \sqrt{T_3^*}}{\sigma_{CA}^* p_{2_i}^*}, \qquad (8)$$

so the air flow rate with fluid injection $\dot{m}_{a_i}$ becomes proportional to $\dot{m}_a$

$$\dot{m}_{a_i} = \dot{m}_a \frac{\pi_{c_i}^*}{\pi_c^*}. \qquad (9)$$

The burned gases flow rate becomes, if one considers the fuel flow rate $\dot{m}_c$,

$$\dot{m}_{g_i} = \dot{m}_{a_i} + \dot{m}_l + \dot{m}_c. \qquad (10)$$

3. ENGINE'S PERFORMANCES

Engine's thrust will, obviously, grow (as figure 2 shows), both because of the flow rate $\dot{m}_{g_i}$ increasing and the specific thrust F_{sp_i} increasing

$$F_i = \dot{m}_{g_i} C_{5_i} - \dot{m}_{a_i} V = \dot{m}_{a_i}\left(\xi_g C_{5i} - V\right), \qquad (11)$$

where C_{5_i} – exhaust gases velocity, V – air-speed, ξ_g – gases mass fraction.

Assuming that the fluid injection is used at low flight speed (only for short time periods during the aircraft take-off maneuver), one can neglect the V – term, so it results

$$F_i = \dot{m}_{a_i} \xi_g C_{5i} = \dot{m}_{a_i} F_{sp_i} = \dot{m}_a \frac{\pi_{c_i}^*}{\pi_c^*} F_{sp_i}. \qquad (11)$$

If the injected fluid is pure (distilled) water its flow rate being at most 4% of the air flow rate, engine's performances (thrust and specific fuel consumption) versus water flow

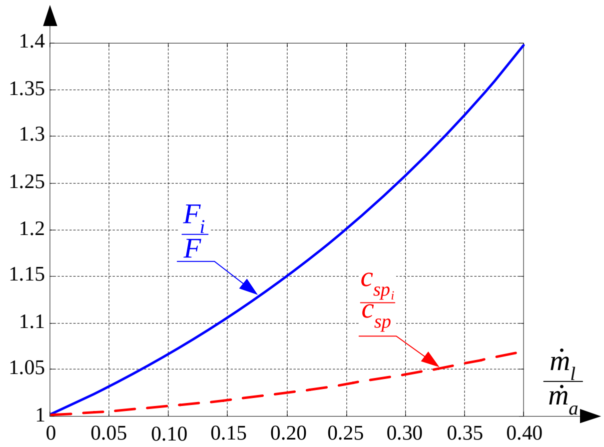

Figure 2. Engine's performances with respect to the injected fluid flow rate

rate fraction are presented in figure 2 [5]. One can observe that for an injected water fraction $\xi_l = \frac{\dot{m}_l}{\dot{m}_a} \le 0.04$, engine's thrust increases up to 40%, which is a real success of the method.

Specific fuel consumption grows too, but moderately, no more than 8%, because of the T_2^* temperature, which decreases and engine's fuel system has to compensate this loss by supplementary fuel injection, in order to restore the requested T_3^* burner's temperature.

If the injection fluid is a combustible substance, such as methanol, or a water-methanol mixture, T_2^* temperature's decreasing and the supplementary need for fuel are partly compensated by the fluid's burning, which brings its own heat into the engine's combustor; consequently, the supplementary fuel injection becomes much smaller than when the injected fluid is pure water.

The described thrust augmentation method is very effective when the engine operates at high atmospheric temperatures (more than 25 ^{o}C), low atmospheric pressures and low humidity (because it facilitates the injected fluid's vaporizing).

One of the method's disadvantage is related to the possibility of air intake's and compressor's blades icing, as well as to the suction of ice lumps into the compressor.

Fluid injection running time is the one who limits the fluid necessary mass to be boarded, so, one has to optimize the on-board fluid supplies, according to the aircraft take-off needs and the atmospheric conditions.

4. ENGINE'S NEW MOTION EQUATIONS

Engine's mathematical model consists of:

- engine's spool motion equation;
- compressor's and turbine's characteristics;
- combustor's energy equation;
- air/gases flow rate's equation.

These equations are studied in [8] for a basic engine; a matrix description is also given

$$[A]\times(u)=(b), \tag{12}$$

where $[A]$ is engine's matrix, $(u)-$ output parameters vector and $(b)-$ input parameters vector:

$$A=\begin{bmatrix} T_1 s+\rho_1 & -k_{1T3} & 0 & -k_{1p2} & k_{1p4} \\ k_{2n} & -k_{2T3} & 0 & k_{2p2} & 0 \\ 0 & -1 & 1 & -k_{3p2} & -k_{3p4} \\ 0 & k_{4T3} & k_{4T4} & k_{4p2} & k_{4p4} \\ k_{5n} & k_{5T3} & 0 & k_{5p2} & 0 \end{bmatrix} \tag{13}$$

$$u^T=\begin{pmatrix} \overline{n} & \overline{T_3^*} & \overline{T_4^*} & \overline{p_2^*} & \overline{p_4^*} \end{pmatrix}, \tag{14}$$

$$b^T=\begin{pmatrix} 0 & 0 & 0 & 0 & k_{5Qc}\overline{Q}_c \end{pmatrix}. \tag{15}$$

The involved co-efficient are used with their expressions, described in [8].

Based on previous chapters thermodynamic considerations, one can affirm that the fluid injection has influence on the compressor's characteristics, as well as on the air/gases flow rate balance along the engine. Consequently, one has to modify the equations involving the compressor's pressure ratio, the pressure and temperature behind the compressor, the temperature behind the engine's combustor, as well as the equation of flow rate's continuity.

One has to study two cases: a) neutral injected fluid (pure water); b) combustible injected fluid (methanol).

One can observe that the flow rate equation is the same for both of these cases, the difference being given by the other equations.

4.1 Flow rate equation. The exhaust gases flow rate is given by Eq. (10), where the fuel flow rate is the smallest and can be neglected

$$\dot{m}_{g_i}\left(p_3^*,T_3^*\right)=\dot{m}_{a_i}\left(p_2^*,n\right)+\dot{m}_l\,. \tag{16}$$

The above equation should be linearised, using the finite differences method, in order to be used into the mathematical model

$$\left(\frac{\partial \dot{m}_{g_i}}{\partial p_3^*}\right)_0 \Delta p_3^* + \left(\frac{\partial \dot{m}_{g_i}}{\partial T_3^*}\right)_0 \Delta T_3^* = \left(\frac{\partial \dot{m}_{a_i}}{\partial p_2^*}\right)_0 \Delta p_2^* + $$
$$+\left(\frac{\partial \dot{m}_{a_i}}{\partial n}\right)_0 \Delta n + \Delta \dot{m}_l\,. \tag{17}$$

Assuming that $p_3^*=\sigma_{CA}^* p_2^*, \sigma_{CA}^*=\text{const.}$, the above equation becomes

$$\left[\left(\frac{\partial \dot{m}_{g_i}}{\partial p_2^*}\right)_0 \sigma_{CA}^* - \left(\frac{\partial \dot{m}_{a_i}}{\partial p_2^*}\right)_0\right]\Delta p_2^* + \left(\frac{\partial \dot{m}_{g_i}}{\partial T_3^*}\right)_0 \Delta T_3^* -$$
$$-\left(\frac{\partial \dot{m}_{a_i}}{\partial n}\right)_0 \Delta n = \Delta \dot{m}_l\,, \tag{18}$$

or
$$\frac{p_{2_0}^*}{\left(\dot{m}_{a_i}\right)_0}\left[\left(\frac{\partial \dot{m}_{g_i}}{\partial p_2^*}\right)_0 \sigma_{CA}^* - \left(\frac{\partial \dot{m}_{a_i}}{\partial p_2^*}\right)_0\right]\frac{\Delta p_2^*}{p_{2_0}^*} +$$
$$\frac{T_{3_0}^*}{\left(\dot{m}_{a_i}\right)_0}\left(\frac{\partial \dot{m}_{g_i}}{\partial T_3^*}\right)_0 \frac{\Delta T_3^*}{T_{3_0}^*} - \frac{n_0}{\left(\dot{m}_{a_i}\right)_0}\left(\frac{\partial \dot{m}_{a_i}}{\partial n}\right)_0 \frac{\Delta n}{n_0} =$$
$$=\frac{\left(\dot{m}_l\right)_0}{\left(\dot{m}_{a_i}\right)_0}\frac{\Delta \dot{m}_l}{\left(\dot{m}_l\right)_0} = \xi_l \frac{\Delta \dot{m}_l}{\left(\dot{m}_l\right)_0}\,. \tag{19}$$

Assuming the formal annotation $\overline{X}=\dfrac{\Delta X}{X_0}$ the above equation becomes

$$k_{2n}^{/}\overline{n} - k_{2T3}^{/}\overline{T_3^*} + k_{2p2}^{/}\overline{p_2^*} = \xi_l \overline{\dot{m}_l}\,; \tag{20}$$

one can observe that the second line in matrix [A] should be replaced by the new values of the co-efficient. Meanwhile, one has to complete the second element in the input vector (b) with the term in the right member of Eq. (20).

4.2 Combustor's energy equation. The fifth equation in the mathematical model is based on the combustor's energy equation,

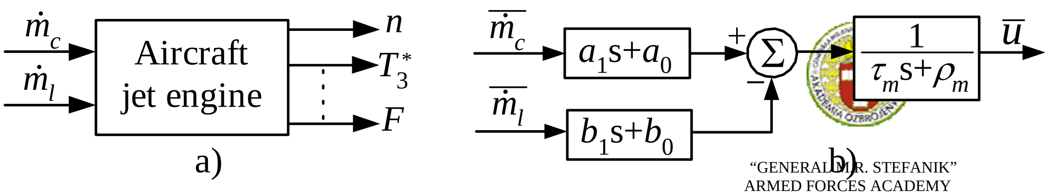

Figure 3. Aircraft jet engine as controlled system (object)

which may have different forms, according to the injected fluid's nature:

$$\dot{m}_{g_i} c_{p_g} T_3^* - \dot{m}_{a_i} c_{p_a} T_2^* = \dot{m}_c \zeta_{CA} P_c + \dot{m}_l \zeta_{CA} P_l \,, \tag{21}$$

where c_{p_g}, c_{p_a} – specific isobar heat of the burned gases and air (assumed as equal), ζ_{CA} – burning process' perfection co-efficient, P_c, P_l – chemical energy of the fuel, respectively of the injected fluid.

Meanwhile, the term T_2^* should be expressed with respect to p_2^*

$$\Delta T_2^* = \left(\frac{\partial T_2^*}{\partial \pi_c^*}\right)_0 \Delta \pi_c^* = \left(\frac{\partial T_2^*}{\partial \pi_c^*}\right)_0 \left(\frac{\partial \pi_c^*}{\partial p_2^*}\right)_0 \Delta p_2^* \,, \tag{22}$$

or as

$$\overline{T_2^*} = \frac{p_{2_0}^*}{T_{2_0}^*} \left(\frac{\partial T_2^*}{\partial \pi_c^*}\right)_0 \left(\frac{\partial \pi_c^*}{\partial p_2^*}\right)_0 \overline{p_2^*} \,. \tag{23}$$

a) If the injected fluid is a neutral-one, the term $\dot{m}_l \zeta_{CA} P_l$ becomes null. Consequently, Eq. (21) becomes

$$\dot{m}_{g_i} c_p T_3^* - \dot{m}_{a_i} c_p T_2^* = \dot{m}_c \zeta_{CA} P_c \,. \tag{24}$$

Considering Eqs. (16), (17), (18) and (23), one obtains from (24)

$$\frac{c_p \left(T_{3_0}^* - T_{2_0}^*\right) n_0}{\dot{m}_{c_0} \zeta_{CA} P_c} \left(\frac{\partial \dot{m}_a}{\partial n}\right)_0 \overline{n} + \frac{c_p \left(\dot{m}_{a_0} - \dot{m}_{l_0}\right) T_{3_0}^*}{\dot{m}_{c_0} \zeta_{CA} P_c} \overline{T_3^*}$$
$$+ \left[\frac{c_p \left(T_{3_0}^* - T_{2_0}^*\right) p_{2_0}^*}{\dot{m}_{c_0} \zeta_{CA} P_c} \left(\frac{\partial \dot{m}_a}{\partial p_2^*}\right)_0 - \frac{c_p \left(\dot{m}_{a_0} - \dot{m}_{l_0}\right)}{\dot{m}_{c_0} \zeta_{CA} P_c} \times \right.$$
$$\left. \times \, p_{2_0}^* \left(\frac{\partial T_2^*}{\partial \pi_c^*}\right)_0 \left(\frac{\partial \pi_c^*}{\partial p_2^*}\right)_0 \right] \overline{p_2^*} = \overline{\dot{m}_c} - \frac{\dot{m}_{l_0}}{\dot{m}_{c_0} \zeta_{CA} P_c} \overline{\dot{m}_l} \,. \tag{25}$$

One can observe that the co-efficient $\frac{\dot{m}_{l_0}}{\dot{m}_{c_0} \zeta_{CA} P_c}$ of the injected fluid flow rate parameter $\overline{\dot{m}_l}$ has a very small value comparing to 1, the value of the co-efficient of $\overline{\dot{m}_c}$, so it may be neglected. Consequently, the above-determined equation may be re-written as

$$k_{5n} \overline{n} + k'_{5T3} \overline{T_3^*} + k'_{5p2} \overline{p_2^*} = \overline{\dot{m}_c} \tag{26}$$

and the last line in matrix $[A]$ should be appropriate restored.

b) If the injected fluid is a combustible-one, the Eq. (21) must be entirely considered. Consequently, the quantity $\dot{m}_l \zeta_{CA} P_l$ adds a new term to Eq. (25) and modifies the co-efficient of $\overline{\dot{m}_c}$, so the right member of the above-mentioned equation becomes

$$\left(1 - \frac{P_l}{P_c} \frac{\dot{m}_{l_0}}{\dot{m}_{c_0}}\right) \overline{\dot{m}_c} - \frac{\dot{m}_{l_0} c_p \left(T_{3_0}^* - T_{2_0}^*\right)}{\dot{m}_{c_0} \zeta_{CA} P_c} \overline{\dot{m}_l} =$$
$$= k'_{5c} \overline{\dot{m}_c} - k'_{5l} \overline{\dot{m}_l} \,. \tag{27}$$

The fifth line's new form is

$$k_{5n} \overline{n} + k'_{5T3} \overline{T_3^*} + k'_{5p2} \overline{p_2^*} = k'_{5c} \overline{\dot{m}_c} - k'_{5l} \overline{\dot{m}_l} \,, \tag{28}$$

so, the mathematical model should be completed. If the second line is multiplied by $\frac{k'_{5l}}{\xi_l}$, then added to the fifth line, one obtains for this one the form

$$\left(k'_{2n} \frac{k'_{5l}}{\xi_l} + k_{5n}\right) \overline{n} + \left(k'_{2T3} \frac{k'_{5l}}{\xi_l} + k_{5T3}\right) \overline{T_3^*} +$$
$$+ \left(k'_{2p2} \frac{k'_{5l}}{\xi_l} + k_{5p2}\right) \overline{p_2^*} = k'_{5c} \overline{\dot{m}_c} \,. \tag{29}$$

The $[A]$-matrix, as well as the (b)-vector should be modified in appropriate modes, with respect to the injected fluid nature.

5. SYSTEM'S QUALITY

Jet engine's behavior, as controlled object (system), should be studied for the new conditions. System's quality consists of engine's step response (its time behavior for step input or inputs).

An aircraft engine with compressor fluid injection can be represented, as controlled object, by a system with two inputs (fuel flow rate and injected fluid flow rate) and more outputs (engine speed, combustor temperature, thrust etc), as figure 3.a shows.

Following the algorithm described in [6,7,8], each one of the outputs u can be expressed with respect to the above-mentioned inputs, as formally shown in figure 3.b.

As main outputs were considered the next three: a) engine's speed non-dimensional parameter $\overline{n}$; b) engine's combustor temperature parameter $\overline{T_3^*}$; c) engine's total thrust parameter $\overline{F}$.

One has chosen, for a quantitative study, a VK-1A-type jet engine, with constant area exhaust nozzle, having in mind only the engine as possible controlled object, without its control systems (without the speed controller and the temperature limiter).

Output parameters' expressions for the VK-1A basic engine are

$$\overline{n}(s)=\frac{1.2606}{2.0859s+5.1015}\overline{\dot{m}_c}, \tag{30}$$

$$\overline{T_3^*}(s)=\frac{1.3799s+2.3888}{2.0859s+5.1015}\overline{\dot{m}_c}, \tag{31}$$

$$\overline{F}(s)=\frac{1.3762s+4.762}{2.0859s+5.1015}\overline{\dot{m}_c}, \tag{32}$$

depicted with continuous lines for step responses in figures 4, 5 and 6.

Figure 4 shows the engine's speed parameters step response, while figure 5 shows the same response of the combustor temperature parameter and figure 6 contains engine's thrust behavior for the same conditions.

The case of the injection of a neutral fluid (pure water) into the compressor brings next mathematical model modifications

$$\overline{n}(s)=\frac{1}{1.6348s+4.5158}\left(1.283\overline{\dot{m}_c}-0.092\overline{\dot{m}_l}\right), \tag{33}$$

$$\overline{T_3^*}(s)=\frac{1}{1.6348s+4.5158}\left[(1.474s+2.748)\overline{\dot{m}_c}-\right.$$
$$\left.-(0.047s+0.034)\overline{\dot{m}_l}\right], \tag{34}$$

$$\overline{F}(s)=\frac{1}{1.6348s+4.5158}\left[(1.385s+4.516)\overline{\dot{m}_c}-\right.$$
$$\left.-(0.089s+0.315)\overline{\dot{m}_l}\right], \tag{35}$$

depicted with dash lines in figures 4, 5 and 6.

The case of the injection of a combustible fluid (methanol) into the compressor brings similar mathematical model modifications, as follows

$$\overline{n}(s)=\frac{1}{2.161s+4.7973}\left(1.43\overline{\dot{m}_c}-0.184\overline{\dot{m}_l}\right), \tag{36}$$

$$\overline{T_3^*}(s)=\frac{1}{2.161s+4.7973}\left[(1.816s+2.467)\overline{\dot{m}_c}-\right.$$
$$\left.-(0.053s+0.047)\overline{\dot{m}_l}\right], \tag{37}$$

$$\overline{F}(s)=\frac{1}{2.161s+4.7973}\left[(1.584s+6.317)\overline{\dot{m}_c}-\right.$$
$$\left.-(0.113s+0.803)\overline{\dot{m}_l}\right], \tag{38}$$

depicted with dash-dot lines in figures 4, 5, 6.

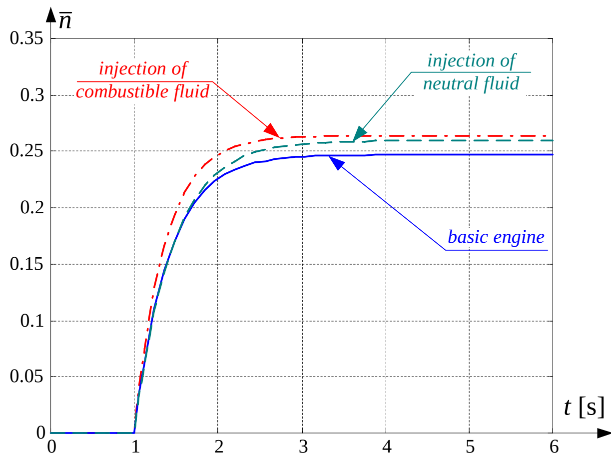

Figure 4. Engine's speed parameter step response

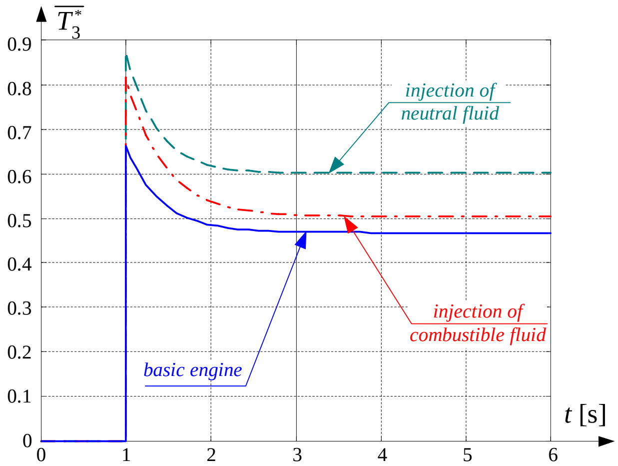

Figure 5. Engine's combustor temperature parameter step response

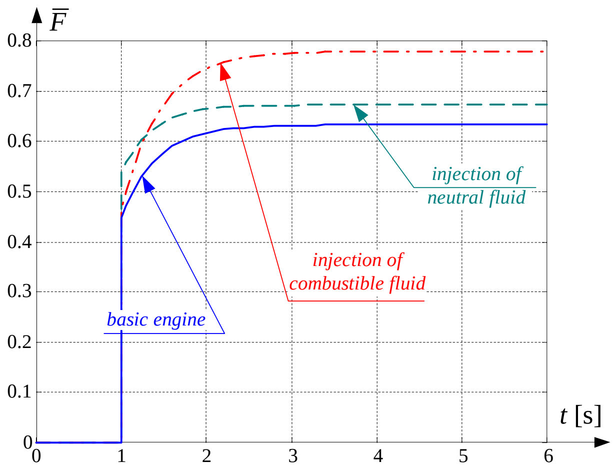

Figure 6. Engine's total thrust parameter step response

6. CONCLUSIONS

Cooling fluid injection into the jet engine's compressor determines gas-dynamic and performance modifications, injected fluid's nature having a very important contribution.

As the technical literature shows, the described method of thrust augmentation is a very effective one, especially for turboprops, thrust increasing being significant; meanwhile, the specific fuel consumption has a moderate growing (under 15%), definitely acceptable because of the thrust augmentation advantages.

Gas-dynamic changes entail both jet engine's mathematical model changes, as well as performances improvements.

Mathematical model's equation were modified because of the air/gases flow rate's balance changes, as well as because of the new energy balance of the combustor, when a combustible cooling fluid is involved.

Whatever the cooling fluid were, engine's speed is less influenced (as figure 4 shows), small $\overline{n}$ – parameter's increasing being observed, especially when the cooling injected fluid is a combustible-one (obviously, because its supplementary contribution to the engine's combustor's burning process and supplementary heat transfer).

From the temperature's parameter behavior point of view (see figure 5), when the injected cooling fluid is a neutral one (e.g. distilled water), one can observe the same trend as for the basic engine, but also a significant growing of $\overline{T_3^*}$ – parameter's value. This is the consequence of the air mass flow rate growing, followed by a supplementary fuel injection.

When the cooling injected fluid is a combustible-one, the initial temperature growing is nearly the same (as for a neutral fluid), but, because of its own heat input, the necessary fuel injection is less than before, so the temperature's parameter tends to restore the basic engine's behavior.

From the thrust-parameter point of view, whatever the injected cooling fluid were, thrust is growing. As figure 6 shows, the combustible cooling fluid injection gives the most significant thrust increase, that because both of air flow rate increase and the specific thrust increase (due to the supplementary heat input). Neutral cooling fluid injection assures thrust increase too, but moderate.

From the engine's time constant point of view, this value remains nearly the same; whatever the method were, stabilization time values are kept around (2.5÷3.0) sec.

One has performed the study for an engine VK-1A-type, for sea level conditions. This study could be extended for other flight conditions (low altitude and take-off air speed), given that this thrust augmentation method is useful for airplane's (aircraft) taking-off, being a restore thrust method (when the atmospheric temperature exceeds a conventionally determined critical value for each engine).

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