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# NONLINEAR EFFECTS OF THE SIDEWASH GRADIENT ON AN AIRPLANE VERTICAL TAIL

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***Abstract:*** *This paper presents an analytical evaluation of the airplane yawing moment, based on the general theory of the aerodynamic design, with details leading first to the separate formulation of the Fourier coefficients in the series solution to the Prandtl's lifting-line theory, followed by the numerical calculation of the sidewash gradient of the vertical tail for a standard configuration of an airplane geometry. Several approaches of the vortex strength and vortex span factors were considered to choose the methodology that better predicts the airplane dynamic handling quality requirements.*

***Keywords:*** *aircraft stability, aerodynamics, Fourier transform*

## 1. INTRODUCTION

A rigid airplane in free flight has three translational and three rotational degrees of freedom. Small translational and rotational disturbances must all result in a return to the original equilibrium attitude. Any object moving through the air will experience drag that opposes the motion. If the angle of attack remains constant, this drag increases with increasing air velocity. The thrust developed by an aircraft engine is either constant with airspeed or decreases with increasing airspeed, so, when an airplane is in static equilibrium with regard to translation in the direction of motion, the forward component of thrust must balance the drag. Any disturbance in velocity in a direction normal to the equilibrium flight path will result in an aerodynamic force that opposes the disturbance. At some angles of attack beyond stall, the airplane must be unstable for disturbances in normal velocity. From an analytical point of view it is easier to design an unmanned aircraft with an autopilot than it is to design a good manned airplane [3].

Static stability in the rotational degrees of freedom is of primary importance for maintaining airplane trim. To be statically stable in rotation, any disturbance in roll, pitch or yaw must all result in the production of a restoring moment that will return the airplane to the original equilibrium attitude [1].

Longitudinal motion involves three degrees of freedom: axil translation, normal translation and rotation in pitch. The other three degrees of freedom, namely sideslip translation, rotation in roll and rotation in yaw are the lateral degrees of freedom which do not compose a planar motion and give the airplane a three dimensional movement. The airplane is defined to have a positive sideslip when the $y_b$ component of airplane velocity relative to the surrounding air is positive. For equilibrium longitudinal motion, the net side force, rolling moment and yawing moment must all be zero. No airplane can be always perfectly symmetric. Asymmetric loading and thrust, propeller rotation or an asymmetric distribution of bugs on the wings can cause either aerodynamic or inertial asymmetry, thus, ever for level flight; some provision must be made for trimming the airplane in roll and yaw. For a standard configuration of airplane geometry, a yaw disturbance in a positive sideslip angle requires a positive yawing moment to restore the disturbance to zero, so, in mathematical terms, static stability in yaw requires that $C_{n,\beta} = \partial C_n / \partial \beta > 0$. Good handling qualities for a typical airplane configuration are normally found with $C_{n,\beta}$ in the range between 0.06 and 0.15 per radian. The size of the vertical tail is not usually fixed by consideration of static stability. When the vertical tail is sized based on control and dynamic handling quality requirements, sufficient static stability is normally provided.

## 2. LATERAL STATIC STABILITY

The yaw stability derivative is estimated by combining the contributions made by the various components of the airplane (fuselage, propeller and vertical tail). The contributions from the fuselage and propeller are typically destabilizing, but these are small and easily countered by the stabilizing effect of an aft vertical tail. The lift developed on the vertical tail as a result of a positive sideslip produces a side force from right to left (fig 1). Since the vertical tail is aft of the airplane center of gravity, this lift produces a positive yawing moment about center of gravity. This is a restoring moment, which tends to point the airplane into the relative wind and return the sideslip angle to zero [4].

If the vertical tail were isolated in a uniform flow field, the angle of attack for this lifting surface would be equivalent to the sideslip angle $\beta$. The magnitude of the airflow relative to the vertical tail can be decreased if the surface is in the wake of the wing or the fuselage.

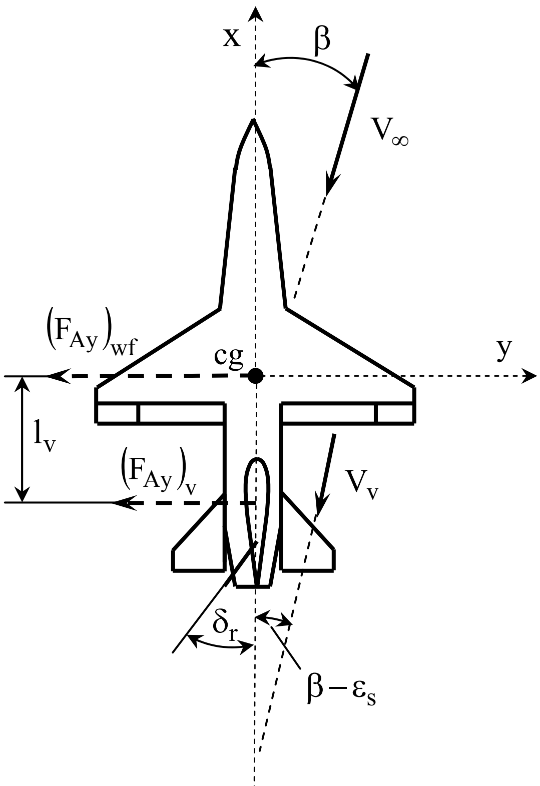


Fig. 1 Effect of sideslip on the aerodynamic yawing moment

This magnitude can be increased if the vertical tail is in the slipstream of a propeller or jet engine [5]. The angle of attack for the vertical tail can be modified by the slipstream of a propeller or by vorticity shed from the main wing. For small sideslip angle $\beta$, the sidewash on a vertical tail can be considered to be a linear function of $\beta$,

$$\varepsilon_s = \varepsilon_{s0} + \varepsilon_{s,\beta} \cdot \beta \qquad (1)$$

where $\varepsilon_{s0}$ is the sidewash angle at zero sideslip and $\varepsilon_{s,\beta}$ is the sidewash gradient,

$$\varepsilon_{s,\beta} = \partial \varepsilon_s / \partial \beta \qquad (2)$$

The lift developed on the vertical tail is linear for small angles of attack and rudder deflection. Positive deflection (leftward) of an aft rudder produces a rightward increment in the lift developed on the vertical tail and a negative increment in the aerodynamic yawing moment for the airplane [2]. Using standard sign convention, the contribution of the vertical tail to the yawing moment is

$$(\Delta n)_v = \frac{1}{2}\rho V_v^2 S_v \cdot \\ \cdot \left[ l_v C_{Lv,\alpha} (\beta - \varepsilon_s - \varepsilon_r \delta_r) + \bar{c}_v C_{mv,\delta_r} \delta_r \right] \qquad (3)$$

where $V_v$ is the local magnitude of the relative airspeed at the position of vertical tail, $C_{Lv,\alpha}$ is the lift slope for the vertical tail, $\varepsilon_r$ is the rudder effectiveness, $\bar{c}_v$ is the mean chord length of the vertical tail and $C_{mv,\delta_r}$ is the change in the moment coefficient for the vertical tail with respect to rudder deflection.

The contribution of the vertical tail to the yawing moment coefficient is

$$(\Delta C_n)_v = \eta_v \frac{S_v l_v}{S_w b_w} C_{Lv,\alpha}\left[(1-\varepsilon_{s,\beta})\beta - C_{Lv,\alpha}\varepsilon_{s0}\right] - \eta_v \frac{S_v l_v}{S_w b_w} C_{Lv,\alpha}\left(\varepsilon_r C_{Lv,\alpha} - \frac{\bar{c}_v}{l_v} C_{m_v,\delta_r}\right)\delta_r \quad (4)$$

where $\eta_v$ is the dynamic pressure ratio for the vertical tail which is analogous to that for the horizontal tail. Equation (4) provides the contributions of the vertical tail and rudder to the yaw control derivative

$$(\Delta C_{n,\beta})_v = \eta_v \frac{S_v l_v}{S_w b_w} C_{Lv,\alpha}(1-\varepsilon_{s,\beta})$$
$$C_{n,\delta_r} = -\eta_v \frac{S_v l_v}{S_w b_w} C_{Lv,\alpha}\left(\varepsilon_r C_{Lv,\alpha} - \frac{\bar{c}_v}{l_v} C_{m_v,\delta_r}\delta_r\right) \quad (5)$$

The aerodynamic derivative $C_{mv,\delta_r}$ is always negative, thus, the change in the airplane yawing moment coefficient with rudder deflection is always negative for an aft rudder, when leftward deflection of the rudder is considered to be positive [6]. The sidewash gradient, $\varepsilon_{s,\beta}$, is typically negative and thus it increases the stabilizing effect of the vertical tail. The lift slope for a vertical tail in combination with a horizontal tail can be estimated using the numerical lifting line method or from three dimensional panel code.

## 3. THE SIDEWASH GRADIENT ON VERTICAL TAIL

The sidewash induced on the vertical tail by the wingtip vortices has a significant effect on the static yaw stability of an airplane. For a vertical tail mounted above the wing, the sidewash gradient is negative and has a stabilizing effect on the airplane. The sidewash gradient produced by the wingtip vortices can be estimated using the vortex model (fig. 2)

According to Biot-Savart law, the y component of velocity induced by the pair of wingtip vortices at the arbitrary point in space $(x, y, z)$ can be written as

$$V_y = \frac{\Gamma_{wt}}{4\pi}\frac{z}{z^2+\left(y+\frac{1}{2}b'\right)^2}\left(1+\frac{x-\frac{1}{2}b'\tan\Lambda}{\sqrt{A}}\right) - \frac{\Gamma_{wt}}{4\pi}\frac{z}{z^2+\left(y-\frac{1}{2}b'\right)^2}\left(1+\frac{x-\frac{1}{2}b'\tan\Lambda}{\sqrt{B}}\right) \quad (6)$$

where

$$A = \left(x-\frac{1}{2}b'\tan\Lambda\right)^2 + z^2 + \left(y+\frac{1}{2}b'\right)^2$$
$$B = \left(x-\frac{1}{2}b'\tan\Lambda\right)^2 + z^2 + \left(y-\frac{1}{2}b'\right)^2 \quad (7)$$

The wingtip vortex strength is proportional to the product of the wing lift coefficient and airspeed. The vortex strength $\Gamma_{wt}$ and spacing $b'$ can be calculated from Prandtl's lifting line theory. Taking into account that the sidewash is positive from left to right (according to the sign convention) applying the small angle approximation, the sidewash angle can be written as

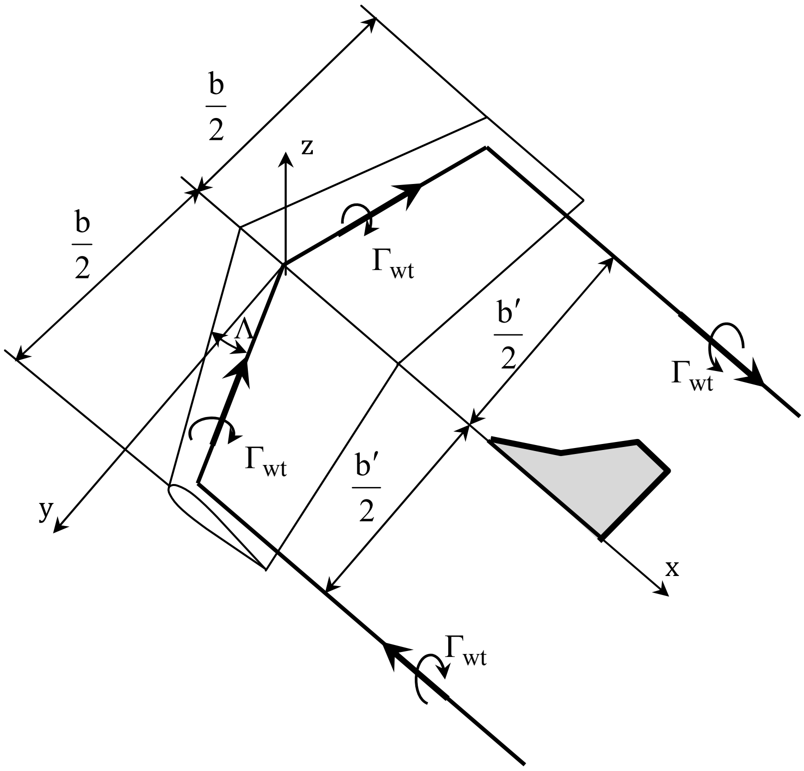


Fig. 2 Vortex model

$$\varepsilon_s = -\frac{V_y}{V_\infty} =$$
$$= \frac{C_{Lw} k_v}{\pi^2 \lambda_w} \frac{\bar{z}}{\bar{z}^2 + (\bar{y} - k_b)^2} \left(1 + \frac{\bar{x} - k_b \tan\Lambda}{\sqrt{\bar{A}}}\right) -$$
$$- \frac{C_{Lw} k_v}{\pi^2 \lambda_w} \frac{\bar{z}}{\bar{z}^2 + (y + k_b)^2} \left(1 + \frac{\bar{x} - k_b \tan\Lambda}{\sqrt{\bar{B}}}\right) \tag{8}$$

where $C_{Lw}$ and $\lambda_w$ are the lift coefficient and aspect ratio for the wing and

$$\bar{x} = \frac{x}{b_w/2},\ \bar{y} = \frac{y}{b_w/2},\ \bar{z} = \frac{z}{b_w/2}$$
$$\bar{A} = (\bar{x} - k_b \tan\Lambda)^2 + \bar{z}^2 + (\bar{y} - k_b)^2$$
$$\bar{B} = (\bar{x} - k_b \tan\Lambda)^2 + \bar{z}^2 + (\bar{y} + k_b)^2$$

The vortex strength factor $k_v$ is a ratio of the wingtip vortex strength to that generated by an elliptic wing having the same lift coefficient and aspect ratio. The vortex span factor $k_b$ is defined as the spacing between the wingtip vortices divided by the wingspan.

When the airplane has some component of sideslip, the wingtip vortices are displaced relative to the position of the vertical tail, as shown in fig. 3.

Using also the small angle approximation, the $y'$ coordinate is related to the y coordinate by the equation

$$\bar{y}'(\beta) = y\cos\beta - \left(x - \frac{1}{2} b' \tan\Lambda\right)\sin\beta \tag{9}$$

Within this small approximation, the sidewash gradient can be written as

$$\frac{\partial \varepsilon_s}{\partial \beta} = \frac{\partial \varepsilon_s}{\partial \bar{y}'} \cdot \frac{\partial \bar{y}'}{\partial \beta} \tag{10}$$

where

$$\frac{\partial \varepsilon_s}{\partial y'} =$$
$$-2\frac{C_{Lw} \cdot k_v}{\pi^2 \lambda_w} \frac{\bar{z}(\bar{y} - k_b)}{\bar{z}^2 + (\bar{y} - k_b)^2} \left(1 + \frac{\bar{x} - k_b \tan\Lambda}{\sqrt{\bar{A}}}\right) +$$
$$+2\frac{C_{Lw} \cdot k_v}{\pi^2 \lambda_w} \frac{\bar{z}(\bar{y} + k_b)}{\bar{z}^2 + (\bar{y} + k_b)^2} \left(1 + \frac{\bar{x} - k_b \tan\Lambda}{\sqrt{\bar{B}}}\right) -$$
$$- \frac{C_{Lw} k_v \bar{z}}{\left[\bar{z}^2 + (\bar{y} - k_b)^2\right]\pi^2 \lambda_w} \frac{(\bar{x} - k_b \tan\Lambda)(\bar{y} - k_b)}{\bar{A}^{\frac{3}{2}}} +$$
$$+ \frac{C_{Lw} k_v \bar{z}}{\left[\bar{z}^2 + (\bar{y} + k_b)^2\right]\pi^2 \lambda_w} \frac{(\bar{x} - k_b \tan\Lambda)(\bar{y} + k_b)}{\bar{B}^{\frac{3}{2}}}$$

and

$$\frac{\partial \bar{y}'}{\partial \beta} = -(\bar{x} - k_b \tan\Lambda) \tag{11}$$

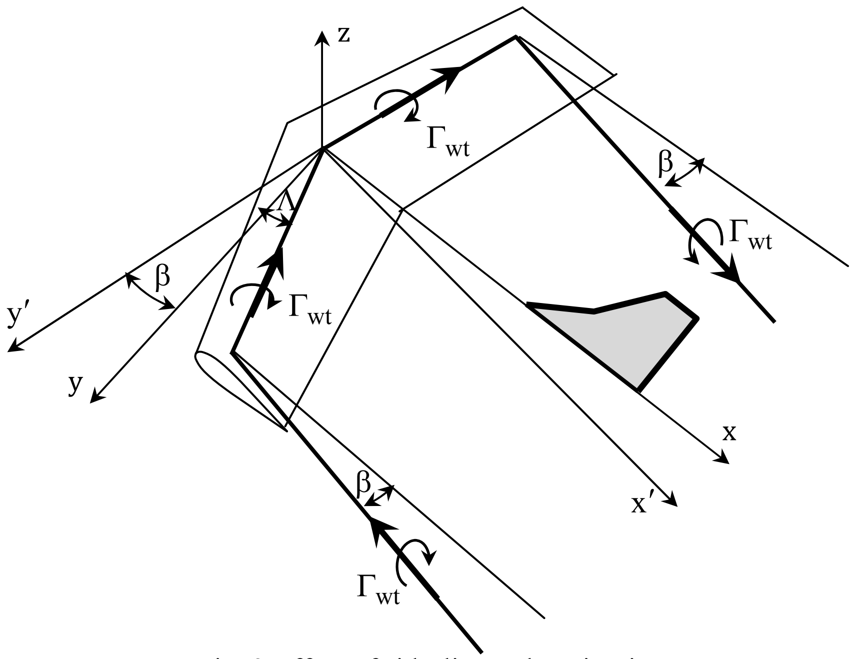


Fig. 3 Effect of sideslip on the wingtip

The sidewash gradient induced at an arbitrary point in space can be estimated by using the above equations. The tail sidewash factor $k_\beta$ depends on the platform shape of the wing and the position of the tail relative to the wing.

## 4. NUMERICAL RESULTS

The parameters $k_v$ and $k_b$ can be written in the form

$$k_v = 1 + \sum_{n=2}^{\infty} \frac{A_n}{A_1} \sin\left(n\frac{\pi}{2}\right)$$

$$k_b = \frac{\frac{\pi}{4} + \sum_{n=2}^{\infty} \frac{nA_n}{(n^2-1)A_1} \cos\left(n\frac{\pi}{2}\right)}{1 + \sum_{n=2}^{\infty} \frac{A_n}{A_1} \sin\left(n\frac{\pi}{2}\right)} \quad (12)$$

where $A_1$, $A_2$,......$A_n$ are the Fourier coefficients in the series solution to Prandtl's Lifting-line equation. The Fourier coefficients for a wing with geometric and aerodynamic twist are given by

$$A_n = a_n(\alpha - \alpha_{L0})_{root} - b_n\Omega \quad (13)$$

where

$$\sum_{n=1}^{N} a_n \left[\frac{2\lambda(1+r)}{\tilde{C}_{L,\alpha}[1-(1-r)|\cos\theta|]} + \frac{n}{\sin\theta}\right] \sin(n\theta) = 1$$

$$\sum_{n=1}^{N} b_n \left[\frac{2\lambda(1+r)}{\tilde{C}_{L,\alpha}[1-(1-r)|\cos\theta|]} + \frac{n}{\sin\theta}\right] \sin(n\theta) = |\cos\theta|$$

$\lambda$ is the aspect ratio, r is the taper ratio, $\tilde{C}_{z,\alpha}$ is the section lift slope for the airfoil from which the wing was generated and $\theta = \cos^{-1}(2y/b)$.

In order to obtain N independent equations for the N unknown Fourier coefficients $a_n$ and $b_n$, the above equations can be written for each span-wise sections of the wing. With the first and last sections located at the wingtips and the intermediate sections spaced equally in $\theta$, this gives the system of equations

$$\begin{bmatrix} A_{11} & A_{12} & \dots & A_{1N} \\ A_{21} & A_{22} & \dots & A_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ A_{N1} & A_{N2} & \dots & A_{NN} \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_N \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} A_{11} & A_{12} & \dots & A_{1N} \\ A_{21} & A_{22} & \dots & A_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ A_{N1} & A_{N2} & \dots & A_{NN} \end{bmatrix} \cdot \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_N \end{bmatrix} = \begin{bmatrix} \cos(\theta_1) \\ \cos(\theta_2) \\ \vdots \\ \cos(\theta_N) \end{bmatrix}$$

where

$A_{1,j} = j^2, \ j = 1, N$

$A_{i,j} =$

$$\left[\frac{2\lambda(1+r)}{\widetilde{C}_{L,\alpha}\left[1-(1-r)\left|\cos(\theta_i)\right|\right]} + \frac{j}{\sin(\theta_i)}\right]\sin(j\theta_i)$$

$\theta_i = \frac{(i-1)\pi}{N-1}, \ i = 2, N; \ j = 1, N$

$A_{N,j} = (-1)^{j+1} j^2, \ j = 1, N$

For a wing platform with an aspect ratio of 8.8, a taper ratio of 0.5 and $S_w = 16\,m^2$, $b_w = 10\,m$, $\lambda = 6.25$, $\widetilde{C}_{Lw,\alpha} = 4.5$, $\Lambda = 10^\circ$ $l_v - l_w = 4\,m$, $\alpha = 5^\circ$, $\alpha_{L0} = -1.5^\circ$, $\overline{x} = 0.9$, and $\overline{z} = 1$, the Fourier coefficients are:

$$a = \begin{bmatrix} 0.198575 \\ 0 \\ 0.006929 \\ 0 \\ 0.009344 \\ 0 \\ 0.003888 \\ 0 \\ 0.003887 \end{bmatrix} \text{ and } b = \begin{bmatrix} -0.011237 \\ 0 \\ 0.008744 \\ 0 \\ 0.011790 \\ 0 \\ 0.004906 \\ 0 \\ 0.004905 \end{bmatrix}$$

The sidewash gradient is

$\varepsilon_{s,\beta} = -0.0043$ .

## 5. CONCLUSIONS

The numerical results show that the sidewash gradient on the vertical stabilizer increases rapidly with the distance aft of the wingtips. Also, there is an optimum in the trade-off between the area of the vertical stabilizer and its distance aft of the center of gravity. The numerical lifting line method gives very good results and can be used for accurately estimating the aerodynamic derivatives associated with tail configurations. The even Fourier coefficients in both $a_n$ and $b_n$ are identically zero for the plane tapered wing, so, the computation time can be reduced by forcing all even coefficients to be zero and solving the system of equations for the old coefficients using sections distributed over only one side of the wing.

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