



"HENRI COANDA"
AIR FORCE ACADEMY
ROMANIA



"GENERAL M.R. STEFANIK"
ARMED FORCES ACADEMY
SLOVAK REPUBLIC

INTERNATIONAL CONFERENCE of SCIENTIFIC PAPER
AFASES 2015
Brasov, 28-30 May 2015

FIGHTING SCHOOL FAILURE IN MATHEMATICS

Monica A.P. Purcaru*, Gabriela Câmpean*

*Faculty of Mathematics and Computer Science, Transilvania University of Brasov, Romania

Abstract: *In the present work, we present examples of several methods to fight school failure in mathematics. In the first paragraph, we define the notion of school failure. In the second paragraph, we deal with the fight against school failure using differentiation and individualization with the help of group interactive methods: Cube, R.A.I., Jigsaw, Brainstorming, Quadrant, Reciprocal learning and teaching. The associated working tools are presented and exemplified with differentiated and individualized teaching activity. In the last paragraph, we analyze and illustrate the role of the ameliorative methods in the fight against school failure in mathematics. Other ways of reducing school failure are being listed and exemplified: interdisciplinary, teaching math games, the use of appropriate teaching materials and e-learning tools.*

Keywords: *e-learning tools, learning difficulties, mathematics, student assessment, teaching methods.*
MSC2010: *97C60, 97C70, 97D40, 97D70.*

1. THE SCHOOL FAILURE IN MATHEMATICS

The Romanian society develops itself primarily through the educational system. In the educational process, its efficiency is of major interest, i.e. school success or failure. It is therefore important to prevent school failure. School failure at mathematics occurs at classes and even at some educational levels. It can lead to a series of future failures in the student's life, while passing through the educational steps successfully will help stimulating and giving him self-confidence.

School failure indicates backwardness in school, un-fulfillment of the mandatory requirements of the educational process, it reflects the discrepancy between the requirements, possibilities and results [3].

School failure is a negative alternative, an un-favorability of school performance [1].

Knowing the individuality of students is an important factor in the prevention, discovery and fight against school failure.

2. FIGHTING SCHOOL FAILURE IN MATHEMATICS USING DIFFERENTIATION AND INDIVIDUALIZATION

A cause of school failure in mathematics is the occurrence of learning difficulties at this subject. One way to reduce or even eliminate these learning difficulties is the differentiation and individualization of work, creating working groups or teamwork.

Differentiated work with students during math lessons, in groups or teams, can be done using some interactive group methods.

Examples:

At the CUBE method, the sides Apply and Describe will contain items of low difficulty, the sides Compare and Associate will contain items of medium difficulty and the sides Analyze and Argue will contain items of increasing difficulty.

At the R.A.I method, the difficulty of the stated question will be directly proportional to the mathematical knowledge of the student who is being asked.

The expert sheet of the JIGSAW method will contain two subthemes having items of reduced difficulty, other two subthemes with items of medium difficulty and the last two subthemes having items of high difficulty.

Using the BRAINSTORMING method, the spontaneity of the given answers will stimulate the creativity of all the students from the classroom.

At the QUADRANT method, the worksheets distributed among the groups of students will be designed differently.

At the RECIPROCAL TEACHING AND LEARNING method, in each group of four: the Questioner will ask clarifying questions for himself, the Summarizer will make a brief summary of the theory read, which will refer to the previous questions, the Clarifier will be required to have an overview and try to answer these questions, while the Predictor will imagine a generalization, or the subsequent course of the theory studied.

Differentiated teaching activity during the mathematics class can be organized using the following appropriate tools: individual worksheets with progressive levels of difficulty: recovery - for students who have difficulties in accumulating knowledge, development - for students with a level higher than average. Also, the homework has to be differentiated, in order to encourage all the students.

Example of differentiated treatment

Sheets of exercises with increasing difficulties, seventh grade – „ Rectangles”

The first sheet includes easier problems, the second consists of medium level problems and the third contains more difficult ones. The teacher guides and helps more the students with fewer opportunities in solving the problems.

Sheet no.1 (Simple problems of calculation)

1. If $AB=5\text{cm}$, $AC=6\text{cm}$, $BC=4\text{ cm}$, then the perimeter of the ABC triangle is.....cm

2. If a field is a square with a side length of 25 m, then the land area is:
a) 625m^2 b) 100m^2 c)
50 m^2 d) $62,5\text{m}^2$

Sheet no.2 (Complex computational problems)

1. A section of the roof in the shape of a trapezoid with bases of 12 m and 6 m and with a height of 4.5 m is covered with tile.

a) Calculate the surface area of the roof section;

b) If for 1 m^2 of roof are needed 8 tiles, how many tiles are needed to cover the roof section?

2. In a triangle with a perimeter of 90 cm, with sides directly proportional to the numbers 3, 5, 6, calculate the triangle side lengths.

Sheet no.3 (The problems of computing which require creativity)

1. The non-parallel sides AB and CD of the trapezoid ABCD intersect in point P.

Knowing that $AB=8$, $AD=12$, $DC=15$, $BC=9$, calculate the lengths of PA, PB, PC, PD.

2. In triangle MNP, $MN=10$, $NP=12$, $PM=8$. Calculate the lengths of the segments determined by the inner bisectors of the opposite sides.

Examples of group-level worksheets (recovery sheet A, development sheet B) - eleventh grade

Topic of the lesson: Applications of determinants in analytic geometry

A

1) Write the equation of the line AB as a determinant and as a general Cartesian form if

a) $A(1,3)$; $B(-2,5)$; b) $A(2,-3)$; $B(-1,5)$; c) $A(2,0)$; $B(0,3)$.

2) Specify which of the points A, B, C are collinear, where: a) $A(-2,5)$, $B(2,3)$, $C(-1,4)$; b) $A(1,1)$, $B(3,3)$, $C(5,5)$; c) $A(-3,2)$, $B(1,2)$, $C(8,2)$.

3) Calculate the area of the ABC triangle where: a) $A(2,1)$, $B(1,5)$, $C(-2,4)$; b) $A(1,1)$, $B(2,2)$, $C(3,4)$; c) $A(3,2)$, $B(6,4)$, $C(9,10)$.

4) Specify which of the points: $A(-3,-3)$, $B(3,6)$, $C(-2,1)$, belong to the line: $3x - 2y + 3 = 0$.



"HENRI COANDA"
AIR FORCE ACADEMY
ROMANIA



"GENERAL M.R. STEFANIK"
ARMED FORCES ACADEMY
SLOVAK REPUBLIC

INTERNATIONAL CONFERENCE of SCIENTIFIC PAPER
AFASES 2015
Brasov, 28-30 May 2015

B

- 1) Calculate the area of the quadrilateral, with edges: A(1,2), B(-3,4), C(-1,-2), D(3,2).
- 2) Demonstrate that ABCD, where: A(0,7), B(6,5), C(5,2), D(-1,4), is a rectangle and then calculate its area.
- 3) Consider ABC triangle, with: A(2,3), B(4,-1), C(5,2) and D(1,2). Demonstrate that the projections of D on the sides of the ABC triangle are three collinear points.
- 4) Consider ABC triangle, with edges: A(-1,-2), B(2,3), C(-1,4). Determine the M point inside the triangle, for which: $S_{AMB} = S_{BMC} = S_{AMC}$.

Also in order to fight school failure in mathematics, individualized activities are very useful and must be carried out with students who have a low level of intellectual development, in order to help them acquire the minimum knowledge required under the program. This type of activity involves adjusting the volume of information and teaching methods to the capabilities of each student.

3. FIGHTING SCHOOL FAILURE IN MATHEMATICS THROUGH AMELIORATIVE MEASURES

Of particular importance in the fight against school failure in mathematics is the taking of ameliorative measures by the teacher, mainly aimed at the correction of essential mistakes for each student, as well as of those typical for the whole class. It is recommended to analyze the mistakes and their ways of reparation in the classroom. It can be suggested to the students to carry out exercises containing typical misconceptions which must be identified.

Examples of misconceptions in mathematics:

Primary school

- there are confusions between the triangle and rectangle or between the inside and outside of a geometrical figure;
- during the transition from oral calculation (in which the numbers to be added up or subtracted are written in line) to written calculation, the students do not correctly arrange the terms/factors in order to perform that operation;
- students make mistakes at the algorithm of calculation of the unknown term;
- students make mistakes at the composing and decomposing of natural numbers, in the recognition of tens digit and the units digit;
- students do not know the neighbors of even and odd numbers;
- students do not command the correct ordering of numbers from least to greatest;
- frequent errors occur at the exercises of addition and subtraction with crossing tens barrier;
- the months of seasons or the own date of birth are not known etc.
- there are difficulties in associating the terminology higher, lower, with the corresponding operation;
- the data of the problem is not being consciously read or analyzed and therefore it is not correctly solved;
- the assignment of the exercise is not carefully read and therefore its requirement is being confused;
- mistakes occur in identifying the first factor of multiplication and in writing the products;
- the expression: "with....more" is confused with „by... times higher”
- mistakes occur in identifying the factors when the product is known;

- difficulties arise at transposing the problem's statement into exercise;
- the association between the name and operation (product, addition, subtraction, ratio) is not correctly made;
- due to inattentive reading of the statement, there are common mistakes at dual-choice items;
- difficulties arise at using the signs $<$, $>$, $=$ in simple inequalities;
- mistakes occur in enunciating a problem according to a literal formula.

Secondary school

- mistakes occur in divisibility problems;
- mistakes occur in exercises with directly and inversely proportional measures;
- difficulties occur at the triangle and circle geometries;
- mistakes occur in applying the short calculation formulas;
- there is not a good command of the rationalization of a fraction;
- there are difficulties in applying the theorem of the three perpendiculars.

High school education

- mistakes occur in exercises with radicals of a degree greater than two;
- mistakes occur in operations with vectors;
- difficulties arise at the demonstration $P(k) \Rightarrow P(k+1)$ for the mathematical induction;
- mistakes occur in calculating the derivatives of composite functions;
- there is not a good command of the application of the Darboux's theorem;
- difficulties arise at problems with recurrent sequences;
- the limits in which the "plier's" theorem is being used are not recognised;
- mistakes occur in the calculation of primitives which use variable change methods;
- mistakes occur in the demonstration of some inequalities using the properties of sequences.

Examples of ameliorative methods in mathematics:

I. At the simulation exam from the eighth grade, half of the pupils pertaining to a class achieved grades below 5. Possible methods to repair this situation are indicated below:

Option no.1- to remedy the observed situation Summative evaluation is usually applied at the end of April. Until the national assessment

date, there is usually an interval of 6-7 weeks. The remediation of the situation described can be achieved in several ways, but a maximum efficiency is given by an intensive additional training program, held at the school outside the class hours. The basic elements of this program are:

1. organizing daily additional training sessions (one hour maximum) on the fundamental topics of algebra and geometry for students who obtained low test grades;
2. each topic of training is accompanied by a short test with 2-3 exercises from the area related to the topic being addressed;
3. each topic must contain a small amount of theory and many examples; it is preferable that the topics which are currently given one or two hours in the class schedule to be divided into smaller sub-themes at the preparatory meetings.
4. the preparation of low-achieving students in mathematics should be done with small steps, and the suggested exercises must be addressed in turn at the blackboard by all the students of the training group;
5. at the end of the week, a summative assessment is applied, consisting of the topics tackled in the current week;
6. at the end of the month, a two-hour test can be applied, consisting of the topics covered in the current month, but, after the completion of the intensive training, a final test is mandatorily applied;
7. all the applied tests are afterwards being punctually discussed with the students; all the problems are being solved on the blackboard by involving those students for whom the solving of those problems was proved difficult or impossible.

Option no.2- to remedy the observed situation

- 1 Investigation through carrying out discussions with students and/or their parents in order to identify their opinion of the poor results obtained.
2. Questionnaires that will include short and clear questions related to the outcome. As all students will answer the same questions, an objective image of the causes which led to the poor results can be created;
3. Systematic observation of student behavior through notation and processing of this data;



"HENRI COANDA"
AIR FORCE ACADEMY
ROMANIA



"GENERAL M.R. STEFANIK"
ARMED FORCES ACADEMY
SLOVAK REPUBLIC

INTERNATIONAL CONFERENCE of SCIENTIFIC PAPER
AFASES 2015
Brasov, 28-30 May 2015

4 Self-assessment tests, in order to evaluate their own level of training.

5. The teacher carries out a remediation plan by identifying common weaknesses in the preparation of students (worksheet with the eventual mistakes; presenting exercises with wrong "solution" and obviously false conclusion and identifying the mistake along with the students; recapitulation and fathoming of notions for the categories of problems and exercises where mistakes have been observed; students need to practice solving different types of tests with various types of items);

6. The teacher discusses with the students all their uncertainties and clarifies what they did not understand.

7. The application of modern methods of teaching and learning such as computer training. Being a very attractive tool, it can help students both at increasing their interest in mathematics and at enhancing their understanding of its contents. Computer assisted learning can contribute to the development of reasoning, imagination and creativity, as well as the ability of self-assessment.

8. Evaluation by computer. Unlike traditional evaluation methods, computer-aided assessment is objective.

9. The use of evaluation sheets. These will include various exercises and problems to be solved by students during the lesson. The evaluation sheet is useful also for the teacher to obtain feedback, upon which he can make suggestions and addenda.

2. The organization mode of the class is explained and the designed tasks are exemplified for a teaching sequence which is aimed at differential treatment, knowing that approximately half of the eighth graders being evaluated do not master the plotting of a first degree function:

- The class is divided into two groups, the group who has assimilated the representation of the function (group I) and the group who has to be assisted in order to represent the function (group II).

- The basic notions related to the first degree function are being repeated: domain, co-domain, representation of a point in the Cartesian system xOy .

- The students from group I are asked to give examples of direct proportionality: distance made-duration, amount-total price etc.

-Students are divided into sub-groups of 3-4 persons Ia, Ib,....., respectively IIa, IIb, ... each subgroup Ia is associated to a subgroup IIa. Those in the first "value" group will have to compose simple exercises for those in group II: axis representation of positive and negative numbers, calculating $f(x)$ for different values of x , representing the pairs of values $(x, f(x))$ in Cartesian system.

-Students in subgroups II have a plenary presentation of solving one of the exercises suggested by group I.

- The level of difficulty increases if there is progress: it moves on to graphical representations of functions, the fields of definition modify or diversify, values of functions are being calculated using graphical methods.

Interdisciplinary work has also triggered good results in the fight against school failure in mathematics, as well as using mathematical teaching games, since they can be used differentially.

Example: (fifth grade) 4 groups are being established: geographers, historians, literates, biologists. Each group receives one sheet with workloads, as follows:

Group 1: Geographers

Alin and Mihai make a trip by following the route Brasov - Bucharest- Constanta. Alin

travels by train and Mihai travels by coach.

Using the table of distances, calculate:

- How many kilometers did Alin make by train;
- How many kilometers did Mihai make by coach;
- Draw the route map of the two boys.

Group 2: Historians

Stefan the Great ruled Moldavia between 1457-1504, while Mircea the Elder ruled Walachia between 1386 -1418.

- For how many years did each of them rule?
- With how many years more than Mircea the Elder did Stefan the Great rule?
- Post on the map the photos of the two rulers in their region of reign.

Group 3: Literates

Using the table, calculate:

- In what year was Mihai Eminescu born?
- In what year was Ion Creangă born?
- If Mihai Eminescu and Ion Creangă died in the same year, by using the data previously discovered, for how many years did each live?

Group 4: Biologists

By calculating the additions and subtractions, you'll find out:

- How many insects can a frog swallow in 24 hours; $458\ 028 - 457\ 728 =$
- How much does the largest frog in the world weigh; $865\ 738 - 165\ 734 =$
- The frog reproduces by laying eggs. Find out how many eggs can a frog release.
 $a - 6\ 748 = 5\ 474$

Also, the systematic use of associated teaching materials will lead to the improvement of outcomes related to mathematics.

Hence, the use of educational software, interactive tables and e-learning platforms train also the low-achieving students in mathematics, and the handling of teaching materials helps them understand the concepts faster.

4. E-LEARNING TOOLS AND THE SCHOOL FAILURE IN MATHEMATICS

Using an e-learning platform for the teaching-learning-testing process has become a modern didactic means nowadays.

If in the mathematics class the professor is using e-learning tools, this will lead to a

increase in students „sympathy” towards this subject, and also to the efficient lesson achievements, this way obtaining progress in mathematics for students with learning difficulties.

Thereby, using the e-learning *AeL Educational* platform as a support for teaching and learning, for testing and assessment, or other interactive blackboards, educational softs, didactic games, assisted by computers, tutorials, as complementary tools instead of using alternative classic methods for teaching will stimulate the interest, the self-confidence and the creativity of students in mathematics. All of this will lead to the removal of backwardness, minimizing the typical mistakes, even obtaining progress at this subject.

5. CONCLUSIONS

By differentiation and individualization, the quality of learning at the mathematics classes is rising. The multitude and variety of the sheets which the students are independently resolving is educating their attention, their capacity of memorizing, this creative imagination and their thinking flexibility. By intensifying individual and differentiate work, weak students obtain satisfaction in tasks accomplishment, gaining confidence in their own strength, and the good ones manifest desire in working additional exercises and increasingly harder problems.

Interactive methods enhance the attention and the interest of students, these ones becoming from spectators, actors. The involvement of the students in resolving group tasks is improving communication and determines development in the group cohesion grade.

Not all the methods are applicable, but the professor needs to learn to adapt and to perfect his theoretical methods depending on the scholar necessities, therefore the creativity of the professor is needed in the lessons success.

All the methods presented in this paper, used separately or altogether, lead to the progress of students who have difficulties in learning mathematics, as they managed to meet the performance standards specified in the curriculum and also to gain self-



"HENRI COANDA"
AIR FORCE ACADEMY
ROMANIA



"GENERAL M.R. STEFANIK"
ARMED FORCES ACADEMY
SLOVAK REPUBLIC

INTERNATIONAL CONFERENCE of SCIENTIFIC PAPER
AFASES 2015
Brasov, 28-30 May 2015

confidence. The joy of success will increase their motivation for learning and effort toward self-improvement, as well as increase their independence in solving the problems, rhythm of work and pleasure to solve additional exercises and problems.

REFERENCES

1. Bontaș, I., *Pedagogie*, Editura ALL, București, 1994, 1998, 2001.
2. Partenie, A. *Să ajutăm corect copiii care întâmpină dificultăți în învățarea matematicii*, Editura Eurobit, Timișoara, 2005.
3. Popescu, V., V., *Succesul și insuccesul școlar – precizări terminologice, forme de manifestare, cauze*, Revista de pedagogie, nr.12, 1991.