LATERAL GUIDANCE LAWS FOR AUTONOMOUS TRACKING FLIGHT PATHS

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Abstract: *Nowadays, for an UAV it is not enough to fly autonomously, so the latest researches in this domain refers to creating unmanned aircraft systems that are able to take decisions by its elves in that concern the obstacles that encounter in their path and finding the most efficient route towards destination. This paper presents three methods for UAV lateral guidance used for tracking straight-line segments and circular orbits. Each of the threes lateral guidance laws are used by path following block to command UAVs whose path following algorithm should be implemented to drive the UAV on the commanded route and to reach the final destination point. It is important to mention that these methods can be used, also, in presence of wind with accurate path following. Combining lateral guidance law with longitudinal guidance law can result a full guidance strategy which can provide the inputs to the autopilot for a very accurate path following.*

Keywords: *UAV, lateral, law, guidance, path following, straight-line, orbit*

1. INTRODUCTION

High computing power available to modern UAVs makes that the commands received by the aircraft to be processed almost in real time. Therefore, this thing led to the emergence of new challenges imposed to UAVs in terms of autonomous flight control. Tracking algorithms for a flight path becomes more and more complex and, also, requires increasingly more computational power.

In terms of an UAV system design, the following path problem is approached after the autopilot have been designed. Therefore, the altitude command, airspeed command, heading command from path following block are sent towards autopilot inputs.

The main path following problem is concerned with the design of control laws that drive an UAV to reach and follow a geometric path. A secondary goal is to force the object moving along the path to satisfy some additional dynamic specification[6].

In the first section of this paper are presented two lateral guidance strategies for straight-line path following. The first one refers to vector field method and the second one describe a path following algorithm depends on lateral acceleration. The notion of vector fields is similar to that of potential fields, which have been widely used as a tool for path planning in the robotics community. Note that the lateral acceleration algorithm is designed to follow a given path. It can be used to fly between waypoints simply by connecting the waypoints with either straight lines or arcs, but additional algorithms are needed to generate the paths and/or waypoints.

In the second section is presented an orbit following algorithm which is implemented to reduce the distance between UAV inertial position and the center of the orbit to be equal to orbit radius.

These lateral guidance laws for autonomous tracking flight paths are often implemented for different configurations of drones, different sizes and different autopilots.

2. LATERAL GUIDANCE STRATEGIES FOR STRAIGHT-LINE PATH FOLLOWING

2.1. VECTOR FIELD METHOD

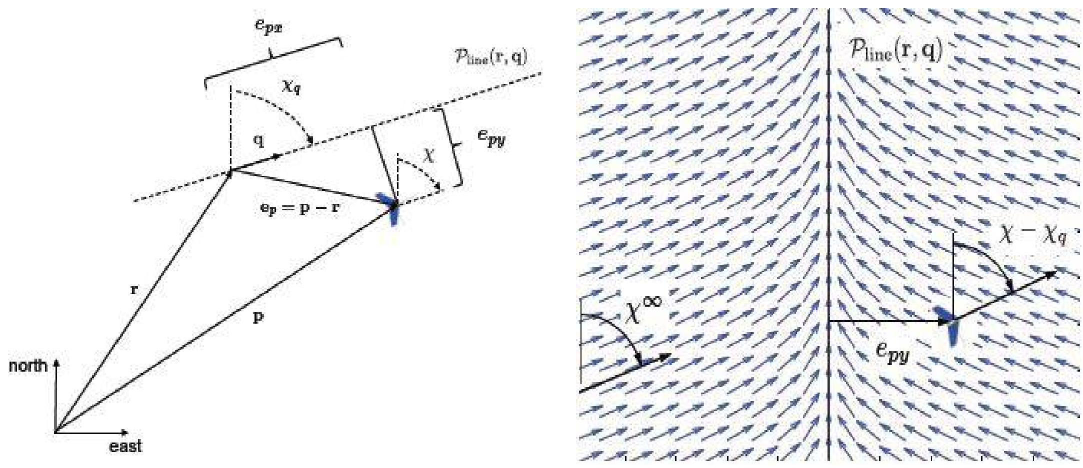
Choosing a vector to be the origin of the path and another one to be a unit vector whose direction indicates the desired direction of travel can be described a straight-line path[1]:

$$P_{line}(r,q)=\{x=r+\lambda q, x\in \mathbb{R}^3, \lambda \in \mathbb{R}\} \tag{1}$$

The course angle of path $P_{line}(r,q)$ as measured from north is given by $\chi_q = \arctan 2\frac{q_e}{q_n}$, where $q=(q_n \quad q_e \quad q_d)^T$ expresses the north, east, and down components of the unit direction vector.

Assume that $\Re_i^{P_{line}} = \begin{pmatrix} \cos\chi_q & \sin\chi_q & 0 \\ -\sin\chi_q & \cos\chi_q & 0 \\ 0 & 0 & 1 \end{pmatrix}$ is the transformation matrix from the inertial frame to the path frame and $e_p = \begin{pmatrix} e_{px} \\ e_{py} \\ e_{pz} \end{pmatrix}$ is the relative path error expressed in the path frame. Therefore, the relative error dynamics in the north-east inertial plane, expressed in the path frame, are given by[1]

$$\begin{pmatrix} \dot{e}_{px} \\ \dot{e}_{py} \end{pmatrix} = V_g \begin{pmatrix} \cos(\chi-\chi_q) \\ \sin(\chi-\chi_q) \end{pmatrix} \tag{2}$$



a) b)

FIG.1. a) The configuration of the UAV indicated by (p,χ), and the straight-line path indicated by $P_{line}(r,q)$; **b)** Vector field for straight-line path following.[1]

The lateral straight-line path following problem is to select commanded course χ_c so that $e_{py} \to 0$ when χ_q is known. The relevant dynamics are therefore given by[1]

$$\begin{aligned} e_{py} &= V_g \sin(\chi - \chi_q) \\ \ddot{\chi} &= b_{\dot{\chi}}(\dot{\chi}_c - \dot{\chi}) + b_\chi(\chi_c - \chi) \end{aligned} \tag{3}$$

The objective for this lateral strategy is to select the commanded course angle χ_c in

equation (3) so that e_{py} is driven to zero asymptotically. The strategy in this section will be to construct a desired course angle at every spatial point relative to the straight-line path that results in the UAV moving toward the path. The set of desired course angles at every point will be called a vector field because the desired course angle specifies a vector (relative to the straight line) with a magnitude of unity. The strategy is to construct the vector field so that when e_{py} is large, the UAV is directed to approach the path with course angle $\chi^\infty \in [0, \frac{\pi}{2})$, and so that as e_{py} approaches zero, the desired course also approaches zero. Thus, the desired course angle of the UAV can be defined as[1]

$$\chi_d(e_{py}) = -\chi^\infty \frac{2}{\pi} \tan^{-1}(k_{path} e_{py}) \tag{4}$$

Where: k_{path} - positive constant that influences the rate of the transition from χ^∞ to zero

Large values of k_{path} result in short, abrupt transitions, while small values of k_{path} cause long, smooth transitions in the desired course.

If χ^∞ is restricted to be in the range $\chi^\infty \in [0, \frac{\pi}{2})$, then clearly

$$-\frac{\pi}{2} < \chi^\infty \frac{2}{\pi} \tan^{-1}(k_{path} e_{py}) < \frac{\pi}{2} \tag{5}$$

for all values of e_{py}.

Therefore in this conditions the Lyapunov function $W(e_{py}) = \frac{1}{2} e_{py}^2$ can be used to argue that if $\chi = \chi_q + \chi_d(e_{py})$, then $e_{py} \to 0$ asymptotically, since

$$\dot{W} = -V_a e_{py}^2 \sin(\chi^\infty \frac{2}{\pi} \tan^{-1}(k_{path} e_{py}))$$

(6)

is less than zero for $e_{py} \neq 0$. The command for lateral path following is therefore given by $\chi_c = \chi_q - \chi^\infty \frac{2}{\pi} \tan^{-1}(k_{path} e_{py})$ (7)

2.2. LATERAL ACCELERATION METHOD

This flight path can be represented as interpolated waypoints or a continuous smooth curve. In either case, this path represents the desired location of the vehicle in 3D.

Accordingly, in FIG.2, using a selected reference point (RF) located on the desired path trajectory can be generated a lateral acceleration command(a_{lat}). This lateral acceleration command can be determinate by

$$a_{lat} = 2\frac{V^2}{L}\sin\eta \tag{8}$$

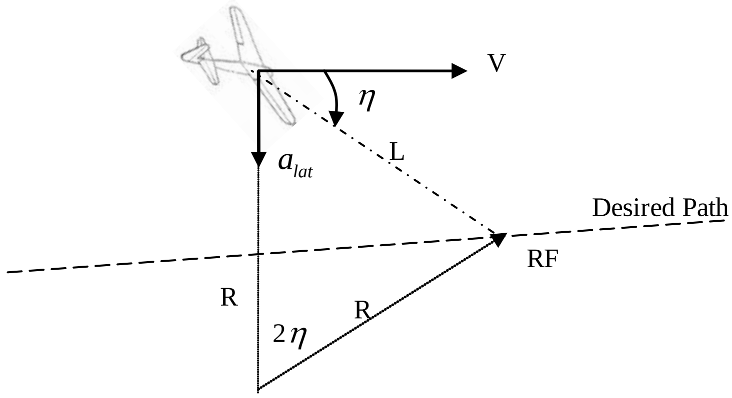


FIG. 2. Guidance algorithm using lateral acceleration

The direction of the lateral acceleration depends of the sign of the angle η. Therefore, if the reference point is to the right side of the UAV, then the UAV tend to align his velocity direction to L line segment direction.

Linearizing the system and assuming that η is small, e is a cross-track error and V is nominal aircraft velocity, the lateral acceleration command can be rewritten[2]:

$$a_{lat} = 2\frac{V}{L}(\dot{e} + \frac{V}{L}e) \tag{9}$$

The guidance law expressed in (8) is very effective and very simple to implement, which makes it to be one of the most used path following method.

Considering ϕ to be the bank angle and assuming that $L\sin\phi = ma_{lat}$ to achieve the desired lateral acceleration and that $L\cos\phi = mg$, then

$$\tan\phi = \frac{a_{lat}}{g} \tag{10}$$

The expression (10) can be used to develop the required bank angle command ϕ_d. Depending on the vehicle, the command ϕ_d is typically saturated at $\pm 15^0$.

3. ORBIT FOLLOWING

A curved path of an object about a point in space is defined as an orbit. Three main parameters can describe the path of the orbit.

$$\Omega_{orbit} = \{o = c + \lambda r(\cos\varphi, \sin\varphi, 0)^T, o \in R^3, \varphi \in [0, 2\pi], \lambda \in \{-1, 1\}\} \tag{11}$$

Where: c – the center of the orbit represented in inertial frame

r – the radius of the orbit

λ - the direction of the orbit ($\lambda = 1$ clockwise orbit, $\lambda = -1$ counterclockwise orbit);

In FIG.3, the distance from the orbit center to the UAV is d, and the angular position of the UAV relative to the orbit is φ.

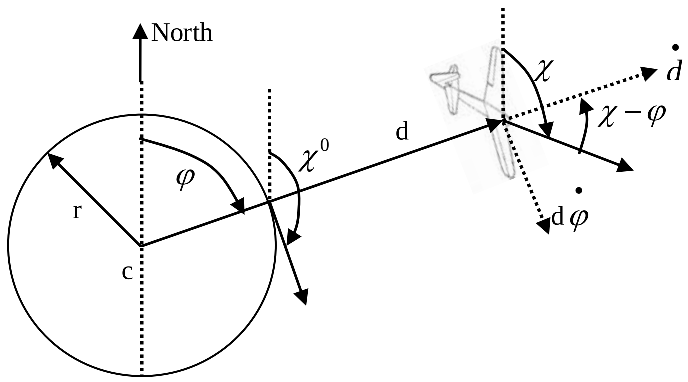


FIG.3. Orbit following algorithm

Assuming that the constant-altitude UAV dynamics in polar coordinates are derived by rotating the differential equations that describe the motion of the UAV in the north and east directions $\dot{p}_n$ and $\dot{p}_e$. Therefore, using the phase angle φ, can be describe the UAV equations of motion in the normal and tangential directions to the orbit:

$$\begin{aligned} \dot{p}_n &= V_g \cos\chi \\ \dot{p}_e &= V_g \sin\chi \end{aligned} \quad \Rightarrow \quad \begin{aligned} \dot{d} &= V_g \cos(\chi-\varphi) \\ \dot{\varphi} &= \frac{V_g}{d}\sin(\chi-\varphi) \end{aligned}$$

Where: V_g represent the ground speed of the UAV.

The control objectives of any orbit following method is to make the distance d similar to the orbit radius ρ and to make the course angle χ to χ^0 in the presence of external disturbances.

Where: $\chi^0 = \varphi + \lambda\frac{\pi}{2}$ - the desired course angle

Like it was shown in section 2.1, it is necessary to construct a course vector field that moves the UAV to the orbit Ω_{orbit}. This course vector field is represented in the following mathematical expression:

$$\chi_d(d-\rho,\lambda) = \chi^0 + \lambda * \tan^{-1}(K_{orbit}(\frac{d-\rho}{\rho})) \tag{12}$$

Where: $K_{orbit} > 0$ is a constant that specifies the rate transition from $\lambda\frac{\pi}{2}$ to zero.

Differentiating the Lyapunov function $W = \frac{1}{2}(d-\rho)^2$ along the system trajectory and assuming that $\chi = \chi_d$ we receive:

$$\dot{W} = -V_g(d-\rho)\sin(\tan^{-1}(K_{orbit}(\frac{d-\rho}{\rho})))$$

(13)

which is negative definite since the argument of sin is in the set $(-\frac{\pi}{2},\frac{\pi}{2})$ for all d>0, implying that $d \to \rho$ asymptotically.

In conclusion the course command for orbit following is given by

$$\chi_c = \varphi + \lambda[\frac{\pi}{2} + \tan^{-1}(K_{orbit}(\frac{d-\rho}{\rho}))]. \quad (14)$$

3. CONCLUSIONS

This paper introduced algorithms for following straight-line paths and circular orbits in the presence of wind. For the first section the idea is to construct a heading field that directs the MAV onto the path and is therefore distinctly different from trajectory tracking, where the vehicle would be commanded to follow a time-varying location.

Methods that I have presented in this paper are much larger debated in [1], [2] and [4]. The purpose of this paper is to describe an integrated approach for developing guidance and control algorithms for autonomous vehicle trajectory tracking.

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