COAXIAL ROTOR SYSTEMS – CHARACTERISTICS AND PERFORMANCES

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Abstract: *In this paper are presented some results regarding the main aerodynamic characteristics and power performances of the coaxial helicopters. Based on the fluid dynamics laws which govern the air flow model through the helicopter rotor disc, in this study were made two case studies which point out that the induced power ratio relative to the power required to operate the two isolated rotors and the coaxial rotors are in favor of the later constructive solution .*

Keywords: *rotor blade, helicopters, autorotation, rotor thrust, induced power.*

1. INTRODUCTION

The main way to distinguish between different helicopter main rotor systems is represented by the blade movement degree of freedom, namely flapping, led-lag and feathering. The most common configuration is the single main rotor helicopter which consists of one main rotor, gearbox, tail rotor drivershaft, intermediate gearbox, tail rotor and engine (fig. 1).

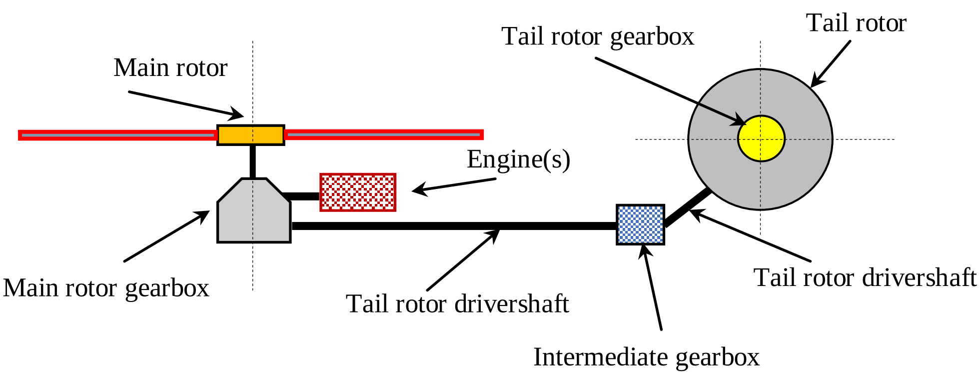


FIG. 1

Another variant is the coaxial rotor helicopter which eliminates the need for a tail rotor by using counter rotating main rotors. One advantage of the counterrotating rotors is that the net size of the rotors is reduced because each rotor provides vertical thrust and all power can provide vertical lift and helicopter control [1]. The tow rotors interact with one another, producing aerodynamic interferences, which leads to loss of system efficiency. Also, this type of helicopter has a very complex mechanical systems, having a great number of moving parts (fig. 2).

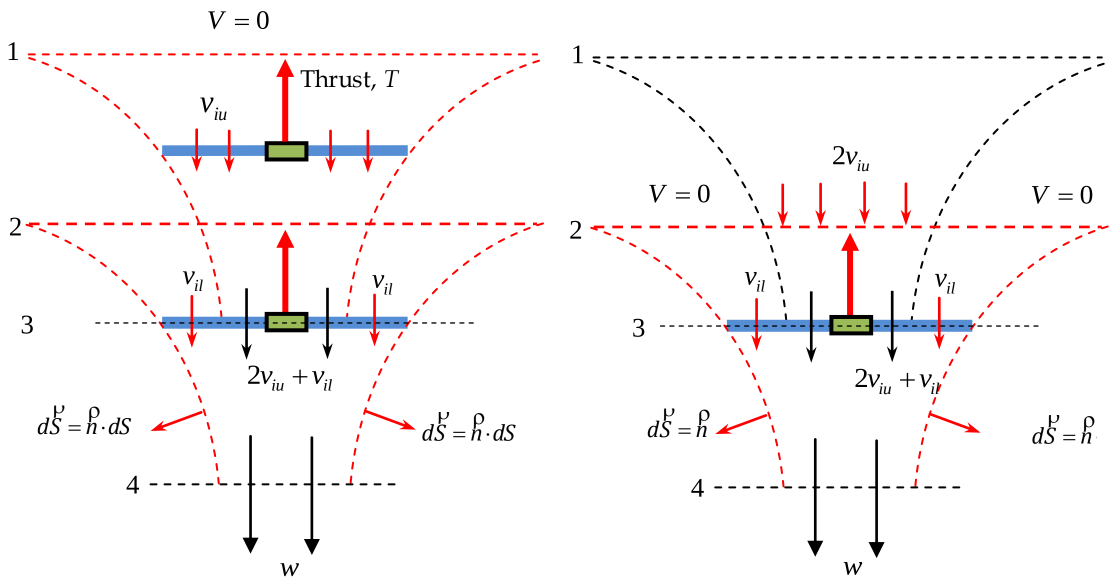
The coaxial rotor systems avoid the effect of lift dissymmetry (lift is proportional to the square of the relative air velocity and it is much greater on the rotor blade advancing side than on the retreating side) [2]. The yaw control is accomplished by increasing the collective pitch of one rotor and decreasing the collective pitch of the other.

Figure 2 shows a coaxial rotors helicopter and the rotors hub with the mechanical links between the swash plates and rotors blades [3].

a) Coaxial rotor   b) Rotor mechanisms

FIG. 2

The flow model for a coaxial helicopter where the lower rotor is considered to operate in the fully developed slipstream of the upper rotor is presented in the fig. 3.



a) Rotor control volume   b) Lower rotor control volume

FIG. 3

In the figure 3 are represented two rotors with the same disc area. The fluid dynamics lows are applied on the whole flow domain [4].

2. INDUCED POWER EVALUATION

Assuming that the rotor planes are sufficiently close together and that each rotor provides an equal fraction of the total system thrust the effective induced velocity of the hole rotor coaxial system is

$$(v_i)_{\substack{independent\\rotors}} = \sqrt{\frac{T}{2\rho A}} \tag{1}$$

where A is the disc rotor area, ρ is air density and T is the rotor thrust.

The induced power is

$$(P_i)_{\substack{independent\\rotors}} = (2T)(v_i)_{\substack{independent\\rotors}} = 2T\sqrt{\frac{T}{2\rho A}} = \sqrt{2}\sqrt{\frac{T^3}{\rho A}} \tag{2}$$

For the coaxial rotors, which together generate a thrust force equal to 2T, the induced velocity has the expression

$$(v_i)_{\substack{coaxial\\rotors}} = \sqrt{\frac{2T}{2\rho A}} \tag{3}$$

The induced power for the coaxial rotors is

$$(P_i)_{\substack{coaxial\\rotors}} = (2T)(v_i)_{\substack{coaxial\\rotors}} = 2T\sqrt{\frac{(2T)}{2\rho A}} = 2\sqrt{\frac{T^3}{\rho A}} \tag{4}$$

If the interference induced power factor k_{int} is considered for the power of the coaxial rotors and the independent rotors, then,

$$k_{\text{int}} = \frac{(P_i)_{\substack{coaxial\\rotors}}}{(P_i)_{\substack{independent\\rotors}}} = \frac{2}{\sqrt{2}} = \sqrt{2} = 1{,}41 \tag{5}$$

which is a 41% increase in induced power relative to the power required to operate the two isolated rotors.

The evaluation of 41% percentage was obtained on the basis of momentum theory without taking into consideration the space between the two rotors. In fact, one of the rotor is placed upper so that the velocity through the lower rotor is two times greater than the velocity through the upper disc rotor [5]. The control volume for coaxial rotors is presented in the fig. 3. Taking into account that the double velocity in the upper current tube is obtained in a section where the area is half of rotor disc area (the two rotors have the same disc area), this means that on a half of the lower rotor the air velocity is $2v_{iu} + v_{il}$ and on the other half the air velocity is v_{il} (fig. 4), where the subscripts u and l have the significance of “upper” and “lower”.

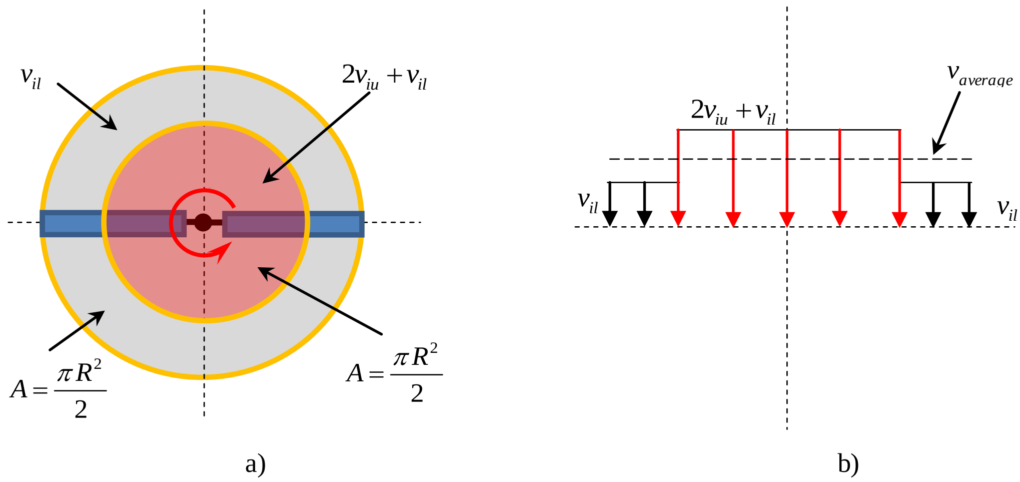


FIG. 4

The flow model applied to the lower rotor is presented in the fig. 3b. According to the momentum equation, the rotor thrust is

$$\vec{T}_l = \iint_{S_4} \rho\left(\vec{V}\cdot d\vec{S}\right)\vec{V} - \iint_{S_2} \rho\left(\vec{V}\cdot d\vec{S}\right)\vec{V} = \dot{m}_l\vec{w} - \dot{m}_u\left(2\vec{v}_{iu}\right) \tag{6}$$

Taking into account the mathematical expressions of the mass flow rates, by replacing it in the above equation, one can get

$$T_l = \rho A\left(v_{iu}+v_{il}\right)w - \rho A v_{iu}\left(2v_{iu}\right) = \rho A\left(v_{iu}+v_{il}\right)w - T_u \tag{7}$$

that leads to the equation

$$T_l + T_l = \rho A\left(v_{iu}+v_{il}\right)w \tag{8}$$

On the lateral surface of the control volume, the double integral is zero because the vectors $d\vec{S}=\vec{n}\cdot dS$ and $\vec{V}$ are perpendicular. Also, the unit vector $\vec{n}$ is oriented outward of the current tube, that is in front of the second integral in equation (6) appears the sign minus, because on this surface and, generally on the any inlet section the air velocity and the unit vector have contrary sign, unlike outlet section, where the air velocity and the unit vector have the same sign [6].

The work on unit time, namely the power consumed by the rotor for gaining in cinetic energy is obtained from the equation,

$$\begin{aligned} P_l &= \iint_{\substack{controll\\surface}} \left(\rho\frac{1}{2}V^2\right)\vec{V}\cdot d\vec{S} = \iint_{suface4}\left(\rho\frac{1}{2}V^2\right)\vec{V}\cdot d\vec{S} + \iint_{\substack{lateral\\surface}}\left(\rho\frac{1}{2}V^2\right)\vec{V}\cdot d\vec{S} + \iint_{surface2}\left(\rho\frac{1}{2}V^2\right)\vec{V}\cdot d\vec{S} = \\ &= \frac{1}{2}\dot{m}_l V^2\Big|_{surface4} + 0 - \frac{1}{2}\dot{m}_l V^2\Big|_{surface2} \end{aligned} \tag{9}$$

The air velocity in section 4 is *w* and in section 2 the velocity is zero in the outside of the upper rotor current tube and $2v_{iu}$ on the inner part of this current tube (fig. 3b), that leads to the following expression for the power P_l,

$$P_l = \frac{1}{2}\big[\underbrace{\rho A\left(v_{iu}+v_{il}\right)}_{\dot{m}_l}\big]w^2 - \frac{1}{2}\big(\underbrace{\rho A v_{iu}}_{\dot{m}_u}\big)\left(2v_{iu}\right)^2 = \frac{1}{2}\rho A\left(v_{iu}+v_{il}\right)w^2 - 2\rho A v_{iu}^3 \tag{10}$$

The power P_l is expressed as a product between the thrust force and the air velocity through the lower rotor disc [7]. The average velocity is obtained as a medium velocity, taking into account that on the inner part of the lower disc the air velocity is $2v_{iu}+v_{il}$ and on the other half part the air velocity is v_{il},

$$v_{average} = \frac{\dfrac{A}{2}v_{il} + \dfrac{A}{2}\left(2v_{iu}+v_{il}\right)}{\dfrac{A}{2}+\dfrac{A}{2}} = v_{iu}+v_{il} \tag{11}$$

Therefore, the lower rotor power consumed has the expression

$$P_l = T_l\left(v_{iu}+v_{il}\right) \tag{12}$$

that leads to the following expression for the energy equation, applied to the lower rotor

$$T_l\left(v_{iu}+v_{il}\right) = \frac{1}{2}\rho A\left(v_{iu}+v_{il}\right)w^2 - 2\rho A v_{iu}^3 \tag{13}$$

or, taking into account the equation (8), it follows that

$$T_l\left(v_{iu}+v_{il}\right) = \frac{1}{2}\left(T_l+T_u\right)w - T_u v_{iu} \tag{14}$$

3. CASE STUDIES

Case 1. The two rotors develop the same thrust force, T

In this situation, the equation (14) becomes,

$$T(v_{iu}+v_{il})=\frac{1}{2}(2T)w-Tv_{iu} \tag{15}$$

and this leads to the following expression for the wake velocity w

$$w=2v_{iu}+v_{il} \tag{16}$$

By replacing velocity w and thrust force $T=2\rho Av_{iu}^2$ in equation (8), one can get $2(2\rho Av_{iu}^2)=\rho A(v_{iu}+v_{il})(2v_{iu}+v_{il})$

(17)

By rearranging the terms and simplifying with ρA, the above equation is transformed in

$$v_{il}^2+3v_{iu}v_{il}-2v_{iu}^2=0 \tag{18}$$

This equation has the following positive solution

$$v_{il}=\left(\frac{-3+\sqrt{17}}{2}\right)v_{iu}=0{,}5616v_{iu} \tag{19}$$

For both rotors the total power is $P_{tot}=P_u+P_l=Tv_{iu}+T(v_{iu}+v_{iu})=T(2v_{iu}+v_{il})=2.5616\,Tv_{iu}$, this means that the induced power factor from interference, k_{int}, is given by

$$k_{\text{int}}=\frac{(P_i)_{\substack{coaxial\\rotors}}}{(P_i)_{\substack{independent\\rotors}}}=\frac{2.5616\,Tv_{iu}}{2Tv_{iu}}=1.281 \tag{20}$$

which is a 28% increase compared to a 41% when the two rotors have no vertical separation.

Case 2. The two rotors develop the same induced power, P.

The mathematical expressions of induced powers for the two rotors are the following:

- The upper rotor, $P_u=2\rho Av_{iu}^3$
- The lower rotor, $P_l=\frac{1}{2}\rho A(v_{iu}+v_{il})w^2-P_u$

From condition that the two induced power are equal, $P_u=P_l=P=2\rho Av_{iu}^3$, it follows that

$$2(2\rho Av_{iu}^3)=\frac{1}{2}\rho A(v_{iu}+v_{il})w^2 \tag{21}$$

or

$$w=\sqrt{\frac{8v_{iu}^3}{v_{iu}+v_{il}}} \tag{22}$$

On the other hand, from the expression of the lower rotor power, $P_l=T_l(v_{iu}+v_{il})$ and mathematical expression of upper rotor power, $P_u=T_uv_{iu}$, it follows also

$$T_l=\frac{P_l}{v_{iu}+v_{il}} \text{ and } T_u=\frac{P_u}{v_{iu}} \tag{23}$$

According to the equation (8), the sum of thrusts is

$$T_u+T_l=\rho A(v_{iu}+v_{il})w \tag{24}$$

so that

$$\frac{P_u}{v_{iu}}+\frac{P_l}{v_{iu}+v_{il}}=\rho A(v_{iu}+v_{il})w \tag{25}$$

By replacing the powers with expression $2\rho A v_{iu}^3$ and the velocity *w* from equation (22) one can obtain

$$2\rho A v_{iu}^3\left(\frac{1}{v_{iu}}+\frac{1}{v_{iu}+v_{il}}\right)=\rho A(v_{iu}+v_{il})\sqrt{\frac{2v_{iu}}{v_{iu}+v_{il}}} \quad (26)$$

With some few transforms and putting the new variable $t = v_{iu}/v_{il}$, the above equation becomes,

$$t\left(1+\frac{1}{1+\frac{1}{t}}\right)=(t+1)\sqrt{\frac{2t}{t+1}} \quad (27)$$

The solution of equation (27) can easily be found in Maplesoft environment (the solution is $t = 2{,}2853$). This means that

$$v_{il}=\frac{1}{2.2853}v_{iu}=0.4375v_{iu} \quad (28)$$

When the coaxial rotors operate at equal rotor torques, the induced power factor k_{int} is given by

$$k_{\text{int}}=\frac{2.4375v_{iu}}{2v_{iu}}=1.219$$

with a 22% increase, compared to the case when the two rotors are operated isolated. Figure 5b shows the power ratio P/P_{hover} for a classical constructive solution (one main rotor and tail rotor configuration).

CONCLUSIONS

The results presented in this paper show that, under some approximation and assumptions, the application of the fluid dynamics laws permits the analysis of the factors that influence the coaxial rotors. The model analyzed in this study allows a preliminary evaluation of the helicopters performances in hover, climb and descent flight.

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