APPLICATIONS OF PROBABILITY THEORY TO STUDY THE INFLUENCE OF PROJECTILE'S MASS TOLERANCES ON EXTERIOR BALLISTIC

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Abstract: *In this paper is presented an algorithm for study the influence of projectile's mass tolerances on projectile exterior ballistic. The algorithm can be useful for engineers who work in design, maintenance or experimental testing of ammunitions. The paper gives an idea of how can be applied the probability theory, in general, and estimation theory in particular to establish the accepted tolerances for projectile's mass considering some exterior ballistic effects imposed.*

Key words: *point-mass trajectory, tolerances, range, deflection, point estimation, confidence intervals estimation*

1. INTRODUCTION

The application of probabilistic calculus gives us the possibility to anticipate in a scientific manner the results for design, maintenance or testing with low resources consumption.

In fact this kind of studies offers to engineers and not only a powerful instrument in evaluate the influence of their choices in: products design, experimental data interpreting or products evaluation in different stages of their lifetime cycle.

Some of these studies are made to evaluate the projectile's point-mass motion in air and evaluate the influence of changes in projectile structure on projectile's point – mass trajectory.

In this paper the study is focused on the evaluation of some main elements of the point-mass projectile's trajectory for an aerodynamic configuration of 30 mm caliber projectile.

The main elements of projectile's point – mass trajectory are calculated considering its mass deviations.

The trajectory's deviations are evaluated based on the calculated main elements of trajectory and also is evaluated the effect to the target caused by the remaining energy of projectile.

The study from this paper is organized in five stages as follows: establish the set of mass values distribution based on its nominal value and tolerances, point – mass trajectory main elements calculation for each value of mass from the set, graphical evaluation of normality distribution of each set of values, point estimation of projectile's mass and trajectory's main elements, trajectory's main elements estimation using confidence intervals.

An example of 30 mm projectile is presented in Fig.1.

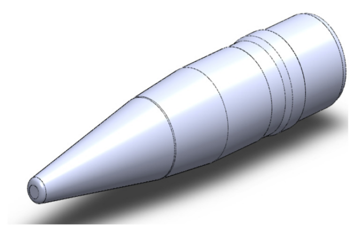

Fig. 1 Example of 30 mm caliber projectile

The study is based on a set of eighteen values for projectile mass, chosen as an example of mass measurement. The main elements of trajectory are calculated using special design software for exterior ballistic calculus for each value from the considered set of values for mass.

An example of projectile's mass tolerances calculus process based on imposed tolerances for range, maximum high, deflection and remaining velocity is presented in Figure 2.

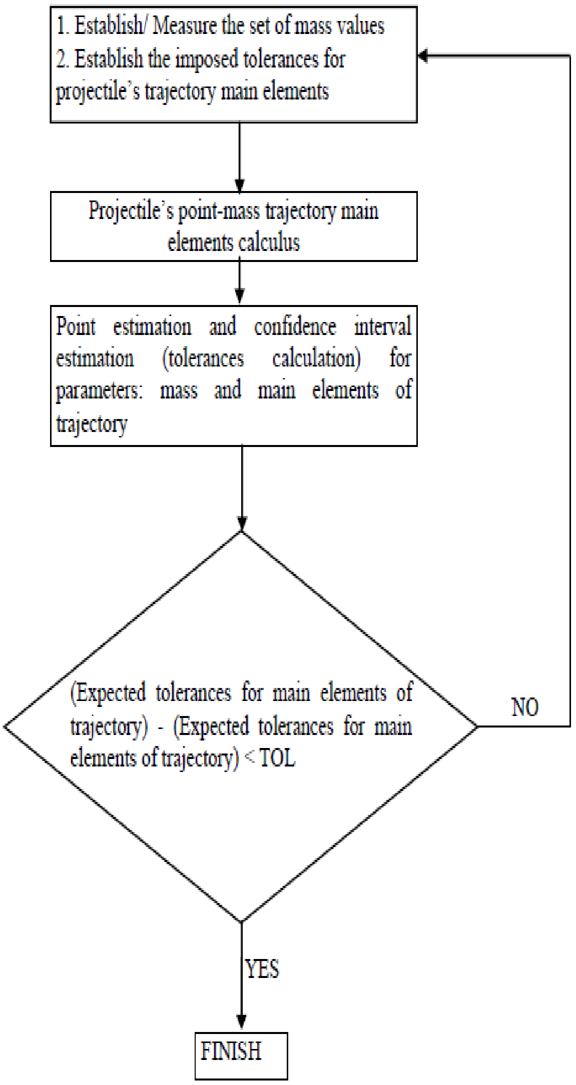

Fig. 2 Diagram of projectile's mass calculus process example

2. MATHEMATICAL MODELS USED

The study has in his five stages the following mathematical models: exterior ballistic model, point estimation for a parameter and parameter tolerances estimation using confidence intervals with a confidence level imposed [2, 3].

The mathematical model for exterior ballistic of projectile point – mass motion assumes to solve the exterior ballistic problem considering 3D point – mass projectile's trajectory (Fig. 3).

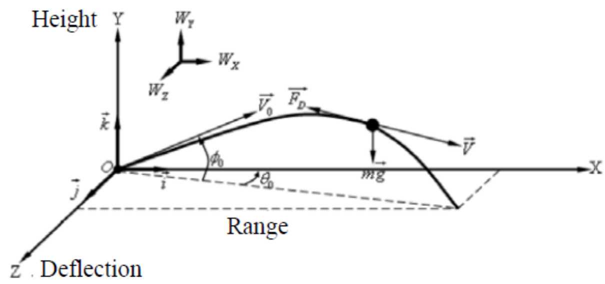

Fig. 3 Point – mass projectile's 3D trajectory

The set of equations used to calculate the 3D trajectory is [1,3]:

$$\begin{cases} \dot{V}_X = -\hat{C}_D \cdot V \cdot (V_X - W_X) \\ \dot{V}_Y = -\hat{C}_D \cdot V \cdot V_Y - g \\ \dot{V}_Z = -\hat{C}_D \cdot V \cdot (V_Z - W_Z) \\ V = \sqrt{(V_X - W_X)^2 + V_Y + (V_Z - W_Z)^2} \end{cases} \quad (1)$$

where $\hat{C}_D = \frac{\rho S C_D}{2m}$, C_D - projectile's drag coefficient, ρ - air density, S - projectile's reference surface, m - projectile's, W_X, W_Z - wind velocity, V_X, V_Y, V_Z - projectile's velocity components, g - gravitational acceleration.

Point estimation and tolerances estimation was made using the following relations [2,3]:

$$x_A \approx \bar{x}_A = \frac{1}{n}\sum_{i=1}^{n} x_i \quad (2)$$

$$\bar{s} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})} \quad (3)$$

$$s = \bar{s} \cdot \sqrt{\frac{n}{n-1}} \quad (4)$$

$$|x_A - \bar{x}_A| < t(P^*, k) \cdot \frac{s}{\sqrt{n}} \quad (5)$$

where x_i - corresponding value from data set, $\bar{x}_A$ - average of data set, x_A - true value, s - empirical standard deviation, $\bar{s}$ - standard deviation, $t(P^*, k)$ - factor calculated using Student repartition, $k = n - 1$, n - number of values from data set.

Using these relations were estimated the tolerances for trajectory's main elements based on projectile's mass tolerances.

3. NUMERICAL RESULTS

Initial data used to calculate the main elements of projectile's point-mass trajectory are presented in Table 1.

Table 1. Initial data for numerical model

Parameter	Value
Caliber [mm]	30
Velocity [m/s]	880
Projection angle [deg]	30
Deflection angle [deg]	0.1
Wind velocity [m/s]	0
Air density [kg/m^3]	1.22
Drag coefficient [-]	0.3053

For projectile's mass were considered eighteen values which includes the average value of 400 grams with values for upper tolerance and lower tolerance of +5 grams and -5 grams.

Numerical results obtained for the set of projectile's mass values are exposed in Table 2.

Table 2. Numerical result for main elements of trajectory

Crt. No.	Mass [g]	Range [m]	Maxim High [m]	Remain Velocity [m/s]
1	395.00	6690.365	1890.609	148.723
2	396.00	6703.164	1893.677	148.887
3	396.75	6713.165	1895.976	149.022
4	397.00	6715.956	1896.742	149.050
5	397.75	6725.952	1899.038	149.185
6	398.25	6732.072	1900.567	149.258
7	398.50	6735.402	1901.332	149.303
8	399.00	6742.062	1902.860	149.393
9	400.00	6754.833	1905.913	149.556
10	401.00	6767.597	1908.962	149.719
11	401.50	6774.249	1910.485	149.809
12	401.75	6777.028	1911.246	149.837
13	402.25	6783.678	1912.767	149.927
14	402.75	6789.778	1914.288	150.000
15	403.00	6793.102	1915.048	150.045
16	403.25	6796.425	1915.808	150.089
17	404.00	6805.843	1918.086	150.207
18	405.00	6819.128	1921.119	150.386

In order to apply the probability theories we made data normality verification for each parameter: mass, range, maxim high and remain velocity. The graphical normality tests are presented in Figures 4 to 7.

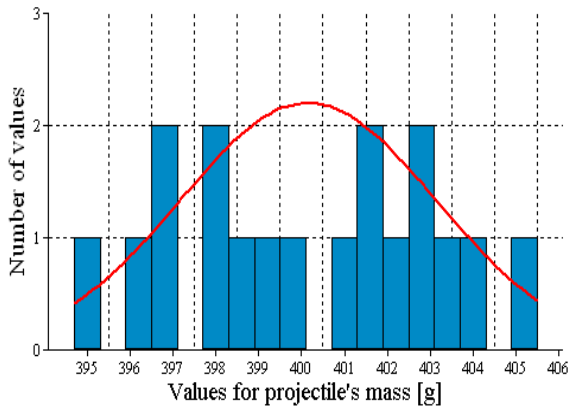

Fig. 4 Normality test results for mass

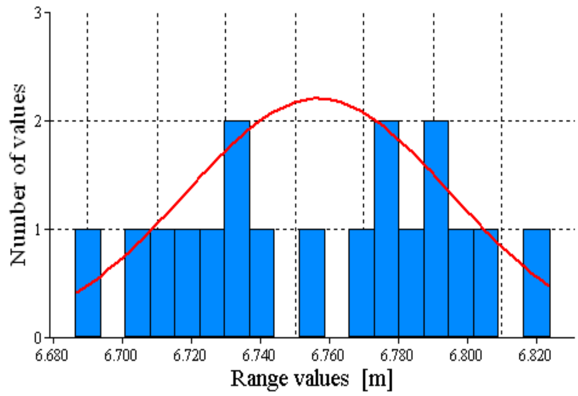

Fig. 5 Normality test results for range

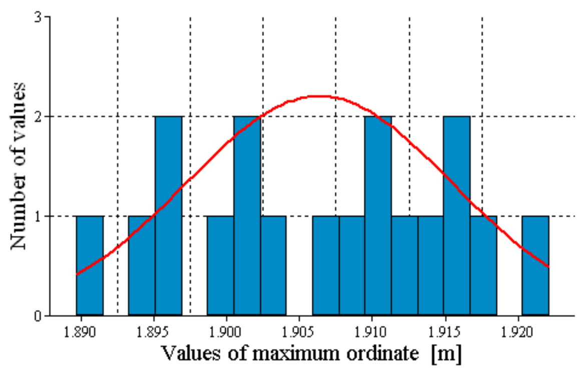

Fig. 6 Normality test results for maxim high

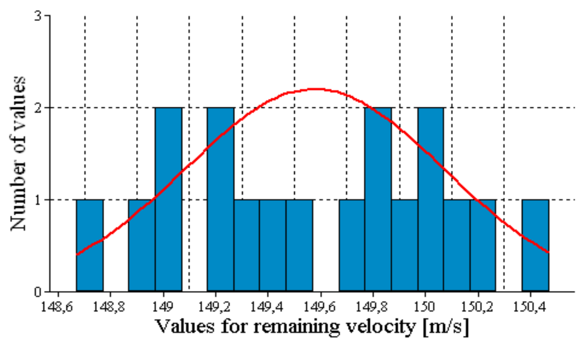

Fig. 7 Normality test results for remain velocity

Once verified the normality distribution of results was calculated the true value (point estimation) and tolerances (confidence intervals) with a confidence level imposed of 99%. The results for mass, range, maxim high and remained velocity are presented in tables 3 to 6.

Table 3. Statistical results mass data

$\overline{x}_A$	s	k	$t(P^*,k)$	$t(P^*,k)\frac{s}{\sqrt{n}}$
400.15	1.65	17	3.01	1.17

Table 4. Statistical results for range data

$\overline{x}_A$	s	k	$t(P^*,k)$	$t(P^*,k)\frac{s}{\sqrt{n}}$
6756.656	5.91	17	3.01	4.19

Table 5. Statistical results for maxim high

$\overline{x}_A$	s	k	$t(P^*,k)$	$t(P^*,k)\frac{s}{\sqrt{n}}$
1906.362	2.88	17	3.01	2.04

Table 6. Statistical results for remained velocity

$\overline{x}_A$	s	k	$t(P^*,k)$	$t(P^*,k)\frac{s}{\sqrt{n}}$
149.578	0.67	17	3,01	0.48

The statistical results offer the possibility to calculate upper and lower tolerances for each parameter: mass, range, maxim high, remained velocity (see Tables 3 to 6)

CONCLUSIONS

The main elements of trajectory were calculated using special design software for exterior ballistic calculus for each value from the considered set of values for mass. An example of projectile's mass tolerances calculus process based on imposed level of trust of 99% was presented.

From this process we can calculate with a 99% confidence the tolerances for range, maxim high, remained velocity for imposed mass tolerances.

The values for calculated parameters are presented in table 7 with tolerances.

Table 7. Tolerances results f

Parameter	Value
Mass [g]	$400.15^{+1.17}_{-1.17}$
Range [m]	$6756.656^{+4.19}_{-4.19}$
Maxim high [m]	$1906.362^{+2.04}_{-2.04}$
Remained velocity[m/s]	$149.578^{+0.48}_{-0.48}$

This kind of study can be extended on any type of parameter not only mass. Also based on probability theory can be determined the need of resources to achieve a result with an imposed level of confidence.

The usefulness of this type of study can be seen in experimental testing, design of different type of products.

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