THE RESONANCE FREQUENCY CORRECTION IN CYLINDRICAL CAVITIES IN AXIAL DIRECTION

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Abstract: *The paper presents a model for determining more precisely the axial resonance frequency in cylindrical resonant cavity. It is known that resonance frequencies on axial direction of the electric field in cylindrical resonant cavity are determined by null points of the Bessel function J_0 but, in practical experiments be noticed a shift in positive direction of the real resonance frequency compared to that calculated. Just this correction is made in this paper.*

Keywords: *electromagnetic field, distribution, Bessel.*

1. INTRODUCTION

To calculate the resonance frequencies in cylindrical and elliptical cavities, it requires a full analysis of electromagnetic (EM) phenomena that happen in this volume. In addition to the Bessel function for field oscillation, the paper presents the analytical field distribution function in opening cavity radius, phenomena taking place simultaneously.

The paper is structured in three parts: deduction of distribution function of electromagnetic field, attaching the J_0 Bessel distribution function, conclusions on the result obtained.

2. DEDUCTION OF DISTRIBUTION FUNCTION OF EM FIELD

For deduction of the distribution function of EM field, is used drawing in FIG. 1 and Maxwell's equations for dynamic field.

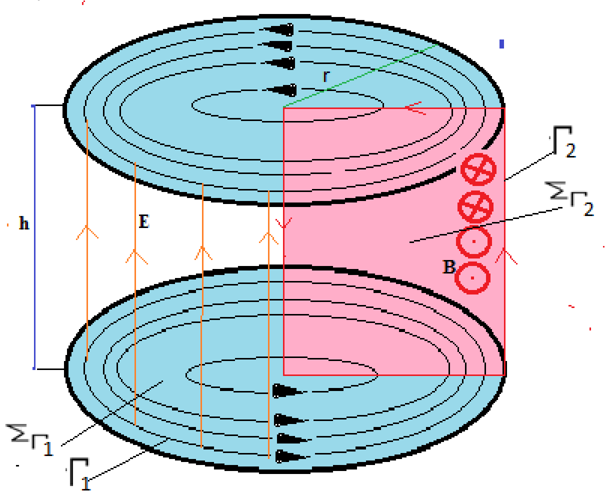

FIG. 1. Schematic representation of the oscillation phenomenon of cavity

$$c^2 \cdot \nabla \times B = \frac{\partial}{\partial t} E \qquad (1)$$

$$\nabla \times E = -\frac{\partial}{\partial t} B \qquad (2)$$

=>

$$c^2 \oint_{\Gamma_1} B\, dl = \frac{\partial}{\partial t} \iint_{\Sigma\Gamma_1} E\, ds_1 \qquad (3)$$

$$\oint E\, dh = -\frac{\partial}{\partial t} \iint B\, ds_2 \qquad (4)$$

$$ds_1 = 2\pi r \cdot dr$$

(I) The first iteration

$$c^2 \cdot 2\pi r\, B_1 = E_0 e^{j\omega t} \cdot j\omega \iint_{\Sigma\Gamma_1} 2\pi r\, dr \qquad (5)$$

$$B_1 = \frac{j\omega}{2c^2} E_0 e^{j\omega t} \cdot r \qquad (6)$$

Writing relation (4) for the contour Γ_1, is obtained:

$$\oint_{\Gamma_1} E_1\, dl = -\frac{\partial}{\partial t} \iint_{\Sigma\Gamma_2} B_1\, ds_2 \qquad (7)$$

$$ds_2 = r \cdot dh$$

Solving integrals, equation (7) becomes:

$$-hE_1 = -\frac{\partial}{\partial t}\left(E_0 e^{j\omega t} \cdot \frac{j\omega}{c^2}\right)\frac{r^2}{2} h \qquad (8)$$

After derivation, follow:

$$-E_1 = E_0 e^{j\omega t} \cdot \frac{\omega^2 r^2}{2c^2} \qquad (9)$$

$$E_1 = -E_0 e^{j\omega t} \cdot \frac{\omega^2}{2c^2} \cdot r^2 \qquad (10)$$

(II) The second iteration

$$c^2 \cdot 2\pi r\, B_2 = \frac{\partial}{\partial t} \iint_{\Sigma\Gamma_1} E_1\, ds_1 \qquad (11)$$

=>

$$c^2 \cdot 2\pi r B_2 = -\frac{j\omega^3}{c^2} E_0 e^{j\omega t} \int r^2 \cdot 2\pi r \, dr \qquad (12)$$

$$B_2 = -\frac{j\omega^3}{2 \cdot 4 \cdot c^4} E_0 e^{j\omega t} \cdot r^3 \qquad (13)$$

$$\oint_{\Gamma_2} E_2 \, dl = -\frac{\partial}{\partial t} \iint_{\Sigma\Gamma_2} B_2 \, ds_2 \qquad (14)$$

=>

$$E_2 = \frac{\omega^4}{2 \cdot 4 \cdot c^4} \cdot E_0 e^{j\omega t} \cdot r^4 \qquad (15)$$

(III) The third iteration

$$c^2 \cdot 2\pi r \, B_3 = \frac{\partial}{\partial t} \iint_{\Sigma\Gamma_1} E_2 \, ds_1 \qquad (16)$$

=>

$$B_3 = \frac{j\omega^5}{2 \cdot 4 \cdot 6 \cdot c^6} E_0 e^{j\omega t} \cdot r^5 \qquad (17)$$

$$\oint_{\Gamma_2} E_3 \, dl = -\frac{\partial}{\partial t} \iint_{\Sigma\Gamma_2} B_3 \, ds_2 \qquad (18)$$

=>

$$E_3 = -\frac{\omega^6}{2 \cdot 4 \cdot 6 \cdot c^6} \cdot E_0 e^{j\omega t} \cdot r^6 \qquad (19)$$

If it continues, is obtained the overall intensity of the electric field E_T as an expression of the form:

$$E_T = E_0 e^{j\omega t} \left[1 - \frac{1}{2}\left(\frac{\omega r}{c}\right)^2 + \frac{1}{2^2}\frac{1}{2!}\left(\frac{\omega r}{c}\right)^4 - \frac{1}{2^3}\frac{1}{3!}\left(\frac{\omega r}{c}\right)^6 + \cdots \right] \qquad (20)$$

If we denote $\frac{\omega r}{c} = x$, we get the expression:

$$E_T = E_0 e^{j\omega t} \left(1 - \frac{1}{2}\frac{1}{1!}x^2 + \frac{1}{2^2}\frac{1}{2!}x^4 - \frac{1}{2^3}\frac{1}{3!}x^6 + \cdots \right) \qquad (21)$$

This relation can be written as:

$$E_T = E_0 e^{j\omega t}\left(1 + \frac{1}{2^i}\sum \frac{x^{2i}}{i!}(-1)^i\right) \tag{22}$$

This is the series expansion of the function:

$$E_T = E_0 e^{j\omega t} e^{-\frac{1}{2}x^2} \tag{23}$$

Or,

$$E_T = E_0 e^{j\omega t} e^{-\frac{1}{2}\left(\frac{\omega r}{c}\right)^2} \tag{24}$$

This expression represents the global electric field intensity distribution, function of the radius cylindrical resonator:

$$E_T = E_0 e^{j\omega t} \cdot f(r) \tag{25}$$

In this expression, for a resonance frequency known,

$$f(r) = e^{-\frac{1}{2}\left(\frac{\omega r}{c}\right)^2} \tag{26}$$

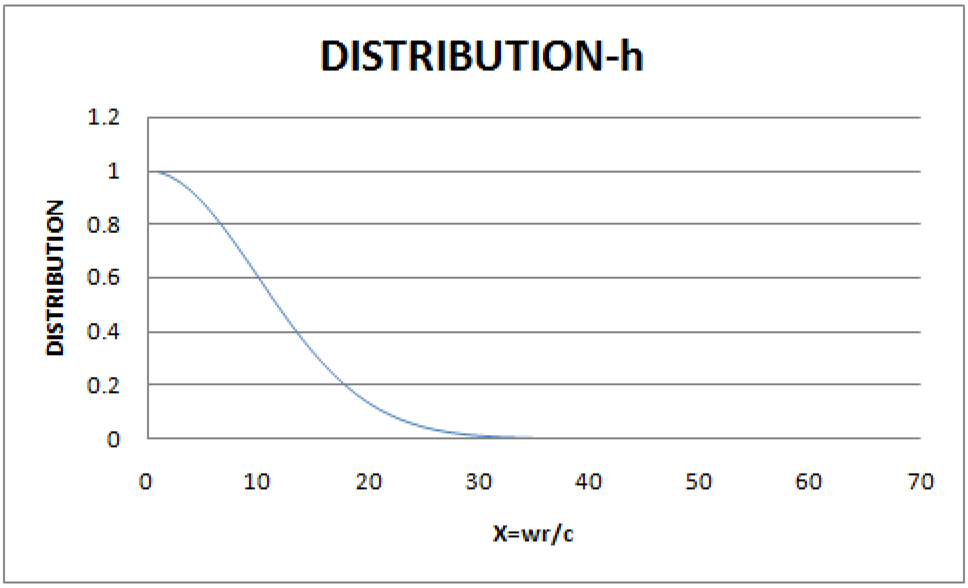

FIG. 2. The distribution of EM field

3. ATTACHING THE J_0 BESSEL DISTRIBUTION FUNCTION

If the calculation is repeated with iterations I, II, III... where is considered that $ds_1 = 2\pi r \cdot dr$ and $ds_2 = r \cdot dh$ (phenomenon development on radius) is obtained the series:

$$E = E_0 e^{j\omega t}\left[1 - \frac{1}{(1!)^2}\left(\frac{\omega r}{2c}\right)^2 + \frac{1}{(2!)^2}\left(\frac{\omega r}{2c}\right)^4 - \frac{1}{(3!)^2}\left(\frac{\omega r}{2c}\right)^6 + \cdots\right] \tag{27}$$

This expression, in restricted form, is:

$$E = E_0 e^{j\omega t} J_0\left(\frac{\omega r}{c}\right) \tag{28}$$

J_0 is the Bessel function of the first kind and zero order and describes at zero crossings, the frequencies around which is carried out the resonance phenomenon of the cylindrical cavity. In practical measurements it has been observed that the real resonance frequencies are shifted slightly to the right compared to the zero crossings of Bessel function,

$J_0(X)|_{X=\frac{\omega r}{c}}$.

This frequency deviation, *Δf,* results from overlapping the function E_T over the function E, the two components of the electric field intensity taking place simultaneously.

$$E_{resultant} = \frac{E_T + E}{2} = \frac{1}{2} E_0 e^{j\omega t}\left(J_0(X) + e^{-\frac{1}{2}X^2}\right) \tag{29}$$

Thus, the correction function is:

$$P(X) = J_0(X) + e^{-\frac{1}{2}X^2} \tag{30}$$

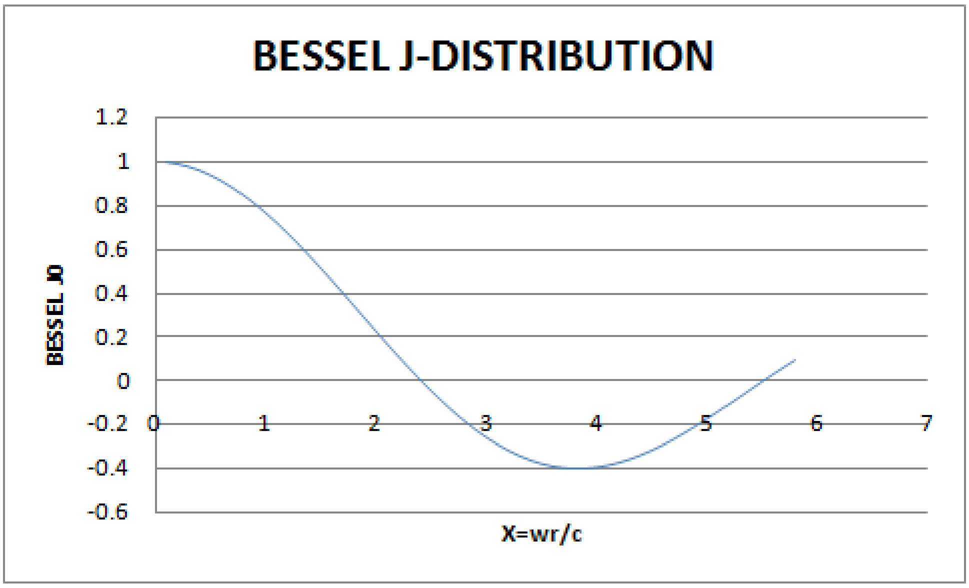

FIG. 3. The Bessel function J_0.

3. CONCLUSIONS

The diagrams in FIG. 2 and FIG. 3 represents the EM field radius distribution, respectively the Bessel function J_0, with independent action. In the diagram in FIG. 4, the Bessel function J_0 and the correction function *P(X)* are simultaneously represented. This figure shows the displacement frequency correction (zero-crossing of *P(X)*) compared to Bessel function, in this way:

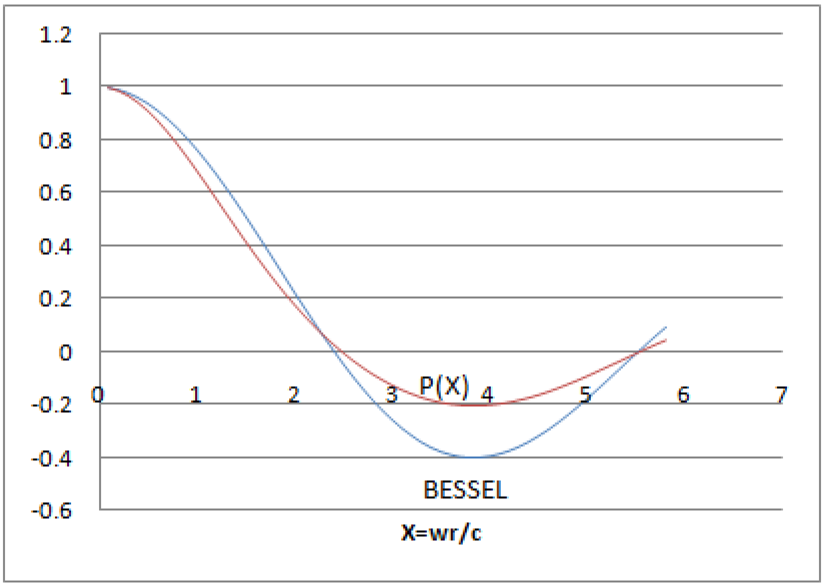

FIG. 4. The Bessel function J_0 and the correction function P(X)

The first null of Bessel function is x = 2.405.
The first null of *P(X)* function is x = 2.47.
The second null of Bessel function is x = 5.52.
The second null of *P(X)* function is x = 5.543.

The resonance frequencies obtained from the correction function *P(X)* have better accuracy with an order of magnitude compared to Bessel function.

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