ON A PARTICULAR LIFETIME DISTRIBUTION

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Abstract: *The paper introduces a probability distribution as a mixture between a Gamma(0, $\lambda\eta$, ν) distribution and an exponential Exp(μ) distribution of η. The first and second order moments are calculated, together with the variance. Algorithms for simulation of the introduced distribution are presented. These include the inverse method and the rejection method. The last section discusses an application to reliability of a system with n components, with stochastic independent lifetimes, namely the distributions of maximum W and minimum V of lifetimes, when components have the introduced distribution. Simulation of W and V is also presented. The last part of the paper presents distributions of V and W when the number of components is (truncated) random with distributions: Poisson(λ), Geometric(p), or Binomial(n, p), $n \geq 1$. Simulation of these distributions is also underlined.*

Keywords: *Gamma and Exponential distributions, Mixture distribution, Random variate simulation, Reliability.*

1. INTRODUCTION

In reliability theory, an important notion is the *lifetime*, i. e. a random variable *rv* which represents the running of a system until it *fails*. Let us denote *L*, a lifetime random variable which has the *cummulative distribution function (cdf)*, $F(x) = P(L < x)$ and the corresponding *probability density function (pdf)*, $f(x) = F'(x)$, assuming that the cdf is a continous (i.e. it is derivable). The *danger of system to fail* is given by the failure rate $r(x)$ defined as [7]

$$r(x) = \frac{f(x)}{\overline{F}(x)} \ , \ \overline{F}(x) - 1 - F(x) + e^{-\int_0^x r(u)du} \tag{1}$$

where $F(x)$ is the *survival probability or reliability function.* The failure rate could be *increasing* i.e. the distribution of *L* is *IFR* (*increasing failure rate*) or is *DFR* (*decreasing failure rate*). As many real systems become *fatigue in time*, then many of reliability systems are IFR. (While, for instance, from reliability point of view, the lifetime of a computer program is DFR!, (see [7]). Examples of IFR (as well as DFR) cdf's are [7] the exponential distribution *Exp*(λ), of parameter λ; $\lambda > 0$; or a *Weibull*(0;λ; υ); λ>0; υ>1 distribution, while when $0<\upsilon<1$, this distribution is DFR. Note that for any lifetime, the *pdf* $f(x)$, as well as *cdf* $F(x)$ is zero for $x \leq 0$*:* Therefore, in the formulae like this, in the following we will specify the pdf $f(x)$ and the cdf $F(x)$, only for $x>0$.

Some time, *complex reliability systems* have a behavior which assumes at the beginning of their life an increasing failure rate and later on, a decreasing failure rate. In this situation are (see [7]) the *lognormal distribution LN*(μ;σ), μ>0; σ>0 and the *Gamma*(0;λ;υ), λ>0; υ>1 *distribution*, which has the pdf

$$f(x) = \frac{\lambda^{\nu}}{\Gamma(\nu)} x^{\nu-1} \; e^{-\lambda x} \; , \;\; x > 0 \; , \;\; where \;\; \Gamma(\nu) = \int_0^{\infty} x^{\nu-1} e^{-x} dx \qquad (2)$$

Now, we assume the following situation inspired by [3,6]: the system was produced in a country (or in a *climate*) and it was stated that the lifetime L has a *Gamma*$(0;\lambda;\upsilon)$, $\lambda>0;$ $\upsilon>1$ distribution. If the system is running (or used) in other conditions (i.e in another *climate*), then it is *assumed* that the initial life time distribution is altered, becoming L^{*}; such that this distribution becomes *Gamma*$(0;\lambda\eta;\upsilon)$, where η is a random variable *Exp*(μ): The problem is to determine the pdf of L^{*} which is a *mixture* or a *composition* from *Gamma*$(\lambda\eta;\upsilon)$ with respect to *Exp*(μ) distribution of η: (Note. In [2,3,6], L^{*} is a mixture between *Exp*$(\lambda\eta)$ and *Gamma*$(0;\lambda;\upsilon)$ of η which is a *Loomax distribution*).

2. THE PROBABILITY DISTRIBUTION OF L^{*}

Let us calculate the pdf of L^* as the **mixture** of a *Gamma*$(0;\lambda\eta;\upsilon)$ distribution with the *Exp*(μ) distribution of η: Using the pdf of η, then the pdf of the mixture is

$$f^{*}(x) = \int_0^{\infty} \frac{(\lambda\eta)^{\nu}}{\Gamma(\nu)} x^{\nu-1} e^{-\lambda\eta x} \mu e^{-\mu\eta} d\eta \; .$$

After some calculations we obtain

$$f^{*}(x) = \frac{\mu\lambda^{\nu}}{\Gamma(\nu)} x^{\nu-1} \int_0^{\infty} \eta^{\nu} e^{-\eta(\lambda x+\mu)} \mu e^{-\mu\eta} d\eta$$

which finally gives

$$f^{*}(x) = \frac{\mu\nu\lambda^{\nu}}{(\lambda x+\mu)^{\nu+1}} \; , \; x > 0 \qquad (3)$$

or if we denote $\theta = \dfrac{\lambda}{\nu}$ the final form is

$$f^{*}(x) = \frac{\nu\theta^{\nu} x^{\nu-1}}{(\theta x+1)^{\nu+1}} \; , \; x > 0 \; , \; \theta > 0 \; , \; \nu > 0 \qquad (3')$$

The cdf of L^{*} is calculated as follows

$$F^{*}(x) = \int_0^{x} f^{*}(u) du = \int_0^{x} \frac{\nu\theta^{\nu} u^{\nu-1}}{(1+\theta u)^{\nu+1}} du$$

which is an integral of a bimome type, i.e.

$$I = \int_0^{x} u^{m} \left(a + bu^{n}\right)^{p} du \; ,$$

where a=1; b =θ, m= υ-1; n=1; $p = -\nu - 1 = \dfrac{\nu+1}{s}$ p=-υ-1, s=-1: According to Tchebycheff's conditions [1], this integral is calculated by using the transform

$$z^{s} = z^{-1} = 1 + \theta x \; , \; hence \;\; u = \frac{1}{\theta} \cdot \frac{1-z}{z} \; , \; du = -\frac{dz}{\theta z^{2}}$$

and because $u = 0 \rightarrow z = 1 \; and \; u = x \rightarrow z = \dfrac{1}{1+\theta x}$ it results that

$$F^*(x)=-\int_1^{\frac{1}{1+\theta x}}\nu\theta^\nu\frac{(1-z)^{\nu-1}}{z^{\nu-1}}\frac{dz}{z^2}z^{\nu+1}=\int_{\frac{1}{1+\theta x}}^1\nu(1-z)^{\nu-1}dz\,.$$

By simple calculations it results that

$$F^*(x)=\left(\frac{\theta x}{1+\theta x}\right)^\nu. \tag{4}$$

The moment of order k is calculated as follows

$$m_k=E\left(L^{*k}\right)=\nu\theta^\nu\int_0^\infty\frac{x^{\nu+k-1}}{(\theta x+1)^{\nu+1}}dx\,.$$

This is again a binome integral where $p=-\nu-1=\frac{\nu+1}{s}$, s=is an integer, $n=1$, $m=\nu+k-1$. According to mentioned Tchebytcheff's conditions [1], the integral can be also calculated by using transform $z^s=1+\theta x$, *i.e.* $z^{-1}=1+\theta x$.

Therefore a k-st iteration (with respect to k) of the integral

$$I_k=\int_0^\infty\frac{x^{\nu+k-1}}{(\theta x+1)^{\nu+1}}dx$$

using the specified transform, gives

$$I_k=\frac{1}{\theta^{\nu+k}}\int_0^1(1-z)^{\nu+k-1}z^kdz=-\frac{1}{\theta^{\nu+k}}\frac{1}{\nu+k}(1-z)^{\nu+k}z^k\Big|_0^1+\frac{k}{\theta^{\nu+k}(\nu+k)}\int_0^1(1-z)^{\nu+k}z^kdz \text{ i.e.}$$

$$I_k=\frac{k}{(\nu+k)\theta^{\nu+k}}\int_0^\infty(1-z)^{\nu+k}z^{k-1}dz\,.$$

For k = 1 one obtain

$$I_1=\frac{1}{(\nu+1)\theta^{\nu+1}}\int_0^\infty(1-z)^{\nu+1}dz=\frac{1}{\theta^{\nu+1}(\nu+1)(\nu+2)}\,.$$

Therefore

$$m_1=\frac{\nu}{\theta(\nu+1)(\nu+2)}\,. \tag{5}$$

For k = 2 one obtain

$$I_2=\frac{2}{(\nu+2)\theta^{\nu+2}}\int_0^\infty(1-z)^{\nu+2}zdz=\frac{2}{\nu+2}\frac{1}{\theta^{\nu+2}}\left[-\frac{(1-z)^{\nu+3}z}{\nu+3}\Big|_0^1+\frac{2}{(\nu+3)(\nu+2)\theta^{\nu+2}}\int_0^\infty(1-z)^{\nu+3}dz\right]=$$

$$=\frac{2}{(\nu+2)(\nu+3)(\nu+4)\theta^{\nu+2}}$$

Hence

$$m_2=\frac{2\nu}{\theta^2(\nu+2)(\nu+3)(\nu+4)}\,. \tag{6}$$

Now the variance $\sigma^2=Var\left(L^*\right)$ is calculated as

$$\sigma^2=m_2-m_1^2=\frac{\nu}{(\nu+2)\theta^2}\left[\frac{2}{(\nu+3)(\nu+4)}-\frac{\nu}{(\nu+1)^2(\nu+2)}\right]$$

which is finally

$$\sigma^2 = \frac{\nu}{(\nu+2)\theta^2}\left[\nu^3 + \nu^{2-4\nu+2}\right] > 0. \tag{7}$$

In simulating reliability models which involve this distribution, it is interesting to built up algorithms (see [5,8]) for simulating it, i.e. algorithms for producing sampling values of L^*.

3. SIMULATION OF THE DISTRIBUTION

Such an algorithm is designed to produce a sampling value of L^* and when repeating it *n* times, to obtain a sample L_1^* , L_2^* , … , L_n^* .

In the following, we present simulation methods for L^*.

3.1 The inverse method

The Chintchin's lemma says (see [5,8]):

Lemma. *If a random variable X has the cdf F*(x) *and U is a rv uniformly distributed over* (0*;* 1)*, then the cdf of* $F^{-1}(U)$ *is F*(x)*: (Note that equivalent relation* $U = F^{-1}(X)$ *is valid).*

Note that each computer (i.e. any language) has an algorithm (*generator*) to produce (when is *called*), an uniform random number *U*, and, when calling it next time, it produces *another* uniform random number *U.*

(In other words, successive calls of the generator, produce a sequence of *U's* independent and uniformly distributed). Note also (see [5,8]) that if in an algorithm appears operation 1-*U*, we can use instead *U,* because when *U* is a random number, 1-*U* is also a random number.

From the *Lemma* it results that in other words, to simulate a sampling value of *X;* the following algorithm is derived:

Alhorithm 1.

begin

-*Simulate a random number U uniformly distributed over* (0, 1);

-*Take* $X = F^{-1}(U)$*;*

end

Therefore, the algorithm applied to L^* is:

The inverse algorithm for simulatin L^*

- *Simulate an uniform random number U*;

- *Calculate* $L^* = F^{-1}(U) = \dfrac{1}{\theta}\dfrac{U^{\frac{1}{\nu}}}{1-U^{\frac{1}{\nu}}}$

Note that if *U* is close to 1 (which might happen!), the algorithm fails, therefore we must *reject* a value of *U* for which $1-U^{\frac{1}{\nu}}$ is close *to the zero of the computer involved* and use the *next* random number *U:*

To simplify the writing, in the following when we use the *U*, it is assumed that it is a random number uniformly distributed over (0, 1)*:*

For simulating L^* we will use also the *acceptance-rejection method* which will be called in the following **the rejection method** (see [5,6,8]).

3.2 The rejection method

This method assumes [4,5,8] that we can simulate some *simpler* random variables S_1, $S_2; :::$ until they satisfy *some condition*; the required random variable X is calculated in terms of random variables S_i , $i \in N$ which satisfy the condition.

There are several theorems which lead to rejection methods. We will use the following two theorems.

Theorem 1 [4,5,8,9]. *Assume that random variable X to be simulated, has a pdf* $f(x) \neq 0$, $x \in DD$ *and there is another random variable Y with pdf* $f(x) \neq 0$, $x \in D \subset R$ *which can be simulated, such as* $(\forall) x \in D$, $\frac{f(x)}{h(x)} \leq \alpha$, $\alpha = const$. *If U is uniform* $(0; 1)$ *stochasticaly independent from Y , and they satisfy the condition*

$$U \leq \frac{f(x)}{\alpha h(x)} , \tag{8}$$

then Y has the pdf f(x):

Therefore the algorithm for simulating X is:

Algorithm 2.

repeat

- *generate U uniform* (0*;* 1);
- *generate Y having pdf h*(*x*)

until $U \leq \frac{f(x)}{\alpha h(x)}$

Take X = Y:

The performance of the algorithm is given by the *accepting probability*

$$p_a = \text{Prob}\left(U \leq \frac{f(Y)}{\alpha\, h(Y)} \right) = \frac{1}{\alpha} \tag{8'}$$

therefore it is necessary that $\alpha > 1$. (The value of p_a results from the proof of the theorem). The algorithm is fast if $p_a < 1$, is close to 1.

A rejection algorithm based on this theorem for simulating L^* is obtained if we take as enveloping pdf

$$h(x) = \frac{\theta}{(1 + \theta x)^2}$$

In this case we have the ratio

$$r(x) = \frac{f^*(x)}{h(x)} = \frac{\nu \theta^{\nu-1} x^{\nu-1} (1 + \theta x)^2}{1 + \theta x^{\nu+1}}$$

Since $r(x)$ is an increasing function and positive, it results that

$$\lim_{x \to} r(x) = \nu \theta^{\nu-1} = \alpha .$$

Hence

$$p_a = \frac{1}{\nu \theta^{\nu-1}} \tag{8''}$$

which makes sense if $\nu\theta^{\nu-1} > 1$. The cdf $H(x) = \int_0^x f^*(u)du$ has the inverse

$$Y = \frac{1}{\theta}\left(1 - \frac{1}{\sqrt{U}}\right)$$

and therefore the rejection algorithm is obvious.

The above theorem is also called the *enveloping theorem* because the main assumption of the theorem says that $f(x) \le \alpha h(x)$, i.e. there is a α such as the graph of $f(x)$ is *enveloped* by the graph of $\alpha h(x)$, $x \in D$.

Theorem 2 [4,5,8]. *Assume that the pdf f(x) of the r.v. X to be simulated is in the form*

$$f(x) = c\, r(x)\left(1 - Q(x)\right) \tag{9}$$

where c = const, r(x) is the pdf of a random variable Y and Q(x) is the cdf of a r.v. Z: Then, the rv Y, satisfying condition $Z \ge Y$ with Z and Y independent random variables, has the pdf f(x):

Hence the theorem says that the sampling value *X* is the *accepted Y.*

The resulting rejection algorithm is

Algorithm 3.
repeat
- *simulate Z;*
- *simulate Y independent of Z*;

until $Z \ge Y$;
Take X = Y.
The performance of the algorithm is given by

$$p_a = \text{Prob}(Z \ge Y) = \frac{1}{c}$$

hence it is necessary that $c>1$. (The value p_a results from the proof of the theorem).
There is another form (a kind of **dual** of the theorem 2), let us call it
Theorem 2' in which the pdf is in the form

$$f(x) = c\, r(x) Q(x) \tag{9'}$$

and the condition becomes $Z \le Y$.
Therefore the algorithm deriving from **Theorem 2'** is
Algorithm 3'
repeat
- *simulate Z*;
- *simulate Y independent from Z*;

until $Z \le Y$;
Take X = Y.
We apply this theorem in the form (9') , i.e.

$$f^*(x) = \frac{\nu\theta^\nu x^{\nu-1}}{1+\theta x^{\nu+1}} = c\, r(x) Q(x)$$

where

$$c = \nu \;,\; Q(x) = \left(\frac{\theta x}{1+\theta x}\right)^{\nu-1} \quad r(x) = \frac{\theta}{(1+\theta x)^2} \tag{9''}$$

and $p_a = \frac{1}{\nu}$ (i.e $\nu > 1$is required). The algorithm is obvious and random variables *Z* and *Y* are simulated by the inverse method according to formulae

$$Z = \frac{1}{\theta}\frac{U^{\frac{1}{q}}}{1-U^{\frac{1}{q}}}\ ,\ Y = \frac{1}{\theta}\frac{1-U}{U}\ .$$

The accepting probability is $p_a = \frac{1}{\nu}$ which works if $\nu > 1$, but not very large.

Finally, we note that for simulating L^* the **algoritnms 1 and 2** are prefered. (They are faster!).

4. AN APPLICATION TO RELIABILITY

Assume that a system consists in *n* components having the lifetimes $L_1^*\ ,\ L_2^*\ ,\ \dots\ ,\ L_n^*$ *n* independent and identicaly distributed. In reliability is interesting to consider random variables

$$V = \min_{1\le i\le n} L_i^*\ ,\ W = \max_{1\le i\le n} L_i^* \tag{10}$$

(The lifetime *V* is applied when *all components fail* and the lifetime *W* is applied when *all components run*). It is obvious that cdf's of these rv's are respectively

$$F_v^*(x) = 1-\left(1-F^*(x)\right)^n\ ,\ F_w^*(x) = \left(F^*(x)\right)^n \tag{11}$$

and the corresponding pdf's are [10]

$$f_v^*(x) = nf^*(x)\left(1-F^*(x)\right)^{n-1}\ ,\ f_w^*(x) = f^*(x)\left(F^*(x)\right)^{n-1} \tag{11'}$$

i.e for our rv L^* we have

$$f_v^*(x) = \frac{n\nu\theta^\nu x^{\nu-1}}{(1+\theta x)^{\nu+1}}\left(1-\left(\frac{\theta x}{(1+\theta x)}\right)^\nu\right)^{n-1}\ ,\ f_w^*(x) = \frac{n\nu\theta^\nu x^{\nu-1}}{(1+\theta x)^{\nu+1}}\left(\frac{\theta x}{(1+\theta x)}\right)^{\nu(n-1)} \tag{11''}$$

Simulation of *V* and *W* can be done using directly formula (10) (i.e. calculating *min* and *max* of the simulated sample $L_1^*\ ,\ L_2^*\ ,\ \dots\ ,\ L_n^*$).

Taking into consideration formulae (11'),(11") and theorems 1 and 2, the following concluding theorem is valid

Theorem 3. *The simulation of V and W can be done by using theorem 1 with enveloping density* $f^*(x)$ *or using theorems 2 and 2', noticing that pdf 's* $f_v^*(x) =\ ,\ f_w^*(x)$ *are in the forms (9) or (9') taking into account froms (11").*

The proof is obvious, existing $0 < \alpha < \infty$ (in theorem 1), $r(x)$ is $f^*(x)$ (from (11') or (11")) and $Q(x)$ in theorems 2 or 2' are either $\left(\frac{\theta x}{(1+\theta x)}\right)^{\nu(n-1}$, *or* $\left(1-\left(\frac{\theta x}{1+\theta x}\right)^\nu\right)^{n-1}$.

4.1. The case when n random

In some practical situations, the number of components of the system is a random variable, say N^* , $N^* > 0$ [10]. Possible discrete distributions [10] are *Poisson*(λ); $\lambda>0$, or *Geometric*(p); $0<p<1$; and these distributions are *truncated* on $[1, \infty)$.

In this case the distribution of L^* is called [10] *target distribution.* (In [10] there are used target distributions Weibull and Loomax). Here we will consider also the *new* case when N^* is *Binomial*(*n; p*); $n \in N$, <*p*<1.

Distributions of *V* **and** *W* **when** N^* **is** *Geometric*(*p*). The frequency of *Geometric*(*p*) distribution, truncated on $[1, \infty)$ is

$$P\left(N^* = k\right) = p\,q^{k-1} \;,\; k = 1, 2, \ldots \tag{12}$$

The cdf of *V* is the *mixture*

$$\Phi_v(x) = \sum_{k=1}^{\infty} p\,q^{k-1}\left(1 - \left(1 - \left(\frac{\theta x}{1+\theta x}\right)^v\right)^k\right) \tag{13}$$

which gives

$$\Phi_v(x) = \frac{p}{1-q} - p\sum_{k=1}^{\infty} q^{k-1}\left(1 - \left(\frac{\theta x}{1+\theta x}\right)^v\right)^k = 1 - p\left(1 - \left(\frac{\theta x}{1+\theta x}\right)^v\right)\sum_{k=1}^{\infty} q^k\left(1 - \left(\frac{\theta x}{1+\theta x}\right)^v\right)^k$$

which finally gives

$$\Phi_v(x) = 1 - p\left(1 - \left(\frac{\theta x}{1+\theta x}\right)^v\right)\frac{1}{1 - q\left(1 - \left(\frac{\theta x}{1+\theta x}\right)^v\right)} \tag{14}$$

The rv *V* **from (10) can be simulated by the inverse method** (based on the *inverse* $\Phi_v^{-1}(U)$ of (14)).

Using the truncated distribution (12), the cdf of *W* is

$$\Phi_w(x) = \sum_{k=1}^{\infty} pq^{k-1}\left(\left(\frac{\theta x}{1+\theta x}\right)^v\right)^k = p\left(\frac{\theta x}{1+\theta x}\right)^v \sum_{k=1}^{\infty} q^{k-1}\left(\left(\frac{\theta x}{1+\theta x}\right)^v\right)^{k-1}$$

which gives

$$\Phi_w(x) = p\left(\frac{\theta x}{1+\theta x}\right)^v \frac{1}{1 - q\left(\frac{\theta x}{1+\theta x}\right)^v} \tag{15}$$

The rv *W* **can be simulated also by the inverse method** (use the *inverse* $\Phi_w^{-1}(U)$ of (15)).

Distributions of *V* **and** *W* **when** *N_* **is** *Poisson*(λ) .

The *truncated Poisson*(λ) distribution is

$$P(N^* = k) = \frac{1}{e^{\lambda} - 1}\frac{\lambda^k}{k!}e^{-\lambda} \;,\; k \geq 1 \;. \tag{16}$$

The cdf of *V* is the mixture

$$\psi_v(x) = \frac{e^{-\lambda}}{e^{\lambda} - 1}\sum_{k=1}^{\infty}\frac{\lambda^k}{k!}\left[1 - \left(1 - \frac{\theta x}{1+\theta x}\right)^v\right]^k ,$$

which finaly is

$$\psi_v(x)=\frac{e^{-\lambda}}{e^{\lambda}-1}\left[e^{\lambda}-e^{\lambda\left(1-\left(\frac{\theta x}{1+\theta x}\right)^{\nu}\right)}\right]. \qquad (17)$$

Therefore, the rv *V* can be simulated by the inverse method.

The cdf of *W* is the mixture

$$\psi_w(x)=\frac{e^{-\lambda}}{e^{\lambda}-1}\sum_{k=1}^{\infty}\frac{\lambda^k}{k!}\left(\frac{\theta x}{1+\theta x}\right)^{k\nu}=\frac{e^{-\lambda}}{e^{\lambda}-1}\sum_{k=1}^{\infty}\frac{1}{k!}\left[\lambda\left(\frac{\theta x}{1+\theta x}\right)^{\nu}\right]^k$$

which finaly gives

$$\psi_w(x)=\frac{e^{-\lambda}}{e^{\lambda}-1}\left(e^{\lambda\left(\frac{\theta x}{1+\theta x}\right)^{\nu}}-1\right). \qquad (18)$$

In this case, the rv *W* is also easy simulated by the inverse method.

Simulation of *V* and *W* when *N_* is *Binomial*(*n; p*).

The truncated *Binomial*(*n; p*), $0<p<1$, distribution for $N^*=k$, $k=1,2,\ldots,n$ has the *frequency* function

$$P\left(N^*=k\right)=p\frac{1}{1-q^n}C_n^k p^k q^{n-k},\ q=1-p,\ k=1,2,\ldots \qquad (19)$$

The mixture cdf of *V* in this case is

$$\Phi_v(x)=\sum_{k=1}^{\infty}C_n^k p^k q^{n-k}\left(1-\left(1-\frac{\theta x}{1+\theta x}\right)^{\nu}\right)^k=\frac{1}{1-q^n}\sum_{k=1}^{\infty}C_n^k p^k q^{n-k}\left(1-\left(1-\frac{\theta x}{1+\theta x}\right)^{\nu}\right)^k$$

which finaly gives

$$\Phi_v(x)=1-\frac{1}{1-q^n}\left\{\left[p\left(1-\left(\frac{\theta x}{1+\theta x}\right)^{\nu}+q\right)\right]^n\right\}. \qquad (20)$$

The mixture of *W* with truncated *Binomial*(*n; p*) is

$$\Phi_w(x)=\frac{1}{1-q^n}\sum_{k=1}^{\infty}C_n^k p^k q^{n-k}\left(\frac{\theta x}{1+\theta x}\right)^{\nu k}$$

which finaly gives

$$\Phi_w(x)=\frac{1}{1-q^n}\left\{\left[p\left(\frac{\theta x}{\theta x+1}\right)^{\nu}+q\right]^n-q^n\right\} \qquad (21)$$

Since cdf's (20) and (21) can be easily inversed, the inverse method for simulating *V* and *W* can be applied. Finally, note that the *hypothesis that N_ is binomial is more realistic* for a sistem with *n* components (*n*= fixed), which might have *only* α components runing ($1<\alpha<n$) at a given time.

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