ABOUT RELATIVE UNITS CALCULUS FOR SOME PARAMETERS AUTOMATIC SYSTEMS CIRCUITS

Horia ANDREI*, Costin CEPIŞCĂ, Nicolae JULA***, Ciprian RĂCUCIU*****

*Universitatea Valahia Târgovişte
**Universitatea Politehnica Bucureşti
***Academia Tehnică Militară Bucureşti

***Abstract**: This paper deals with the calculus of the sensitivities in steady state electrical networks using the relative values, which characterize the current and respectively tension harmonics. It is presented a method for computation of the sensitivities correlating it to the apparent power in electrical steady state linear networks. The given example is used for the discussion of some aspects related to the sensitivity problems.*

Key words: harmonics, relative values, sensitivity, transfer functions, apparent power.

1. INTRODUCTION

The sensitivity has been an interesting problem for a long time in the technical literature. The purpose of this paper is to define some sensitivities correlated with the magnitude of input harmonics in steady state linear networks, without knowing the circuit structure.

2. SOME DEFINITIONS OF THE SENSITIVITY IN STEADY STATE LINEAR NETWORKS

Classical sensitivity of a network function F on an independent parameter x is defined [1] by relation:

$$S(F,x)=\frac{x}{F}\frac{\partial F}{\partial x}=\frac{\partial(\ln F)}{\partial(\ln x)} \tag{1}$$

As a rule by m-order sensitivity we understand:

$$S^{(m)}(F,x)=\frac{x^m}{F^m}\frac{\partial^{(m)}F}{\partial x^{(m)}} \tag{2}$$

Let's consider a steady state linear network. To illustrate the relative contributions of all harmonics of current and tension, we can use two formulae equivalents [3] of the Fourier series. If the receptor is linear and supplied with a nonsinusoidal tension u(t) whose the development in a Fourier's series is truncated only at n terms, the characterization of the receptor introducing a φ_k phase angle on each harmonic at the input terminals can be done as follows:

$$u(t)=U\sum_{k=1}^{n}b_k\sqrt{2}\sin(kwt+\alpha_k)=$$
$$=U_1\sum_{k=1}^{n}\mu_k\sqrt{2}\sin(kwt+\alpha_k) \tag{3}$$

$$i(t)=I\sum_{k=1}^{n}a_k\sqrt{2}\sin(kwt+\alpha_k-\varphi_k)=$$
$$=I_1\sum_{k=1}^{n}\varepsilon_k\sqrt{2}\sin(kwt+\alpha_k-\varphi_k) \tag{4}$$

where the relative values of each k - harmonics magnitude of the tension b_k and current a_k as compared to the root-mean-square (RMS) value of the tension, U, respectively current, I, have been taken down as:

$$b_k=\frac{U_k}{U},\quad a_k=\frac{I_k}{I} \tag{5}$$

Otherwise, we can use other the two relative values μ_k the fractions of the magnitude of each k - harmonics of tension compared to the fundamental value U_1 of the tension, respectively, ε_k the fractions of the magnitude of each k - harmonics of current compared to the fundamental value I_1 of the current:

$$\mu_k = \frac{U_k}{U_1}, \quad \varepsilon_k = \frac{I_k}{I_1} \tag{6}$$

These four relative and a-dimensional parameters verify the conditions:

$$\sum_{k=1}^{n} a_k^2 = \sum_{k=1}^{n} b_k^2 = 1 \tag{7}$$

$$b_j^2 = \mu_j^2 / \sum_{k=1}^{n} \mu_k^2, \quad a_j^2 = \varepsilon_j^2 / \sum_{k=1}^{n} \varepsilon_k^2,$$

$$\forall\, j = 1,..., n \in N \tag{8}$$

In this case, for a circuit with two ports, for example: linear electronically amplifiers, linear analogue filter, linear receptors, which has input (i) respectively output (o), we can define two magnitude tension and current transfer function:

$$F_U = \frac{U_0^2}{U_i^2} = \frac{\sum_{k=1}^{n} U_{0,k}^2}{\sum_{k=1}^{n} U_{i,k}^2} = \frac{U_0^2 \sum_{k=1}^{n} b_{0,k}^2}{U_i^2 \sum_{k=1}^{n} b_{i,k}^2} =$$

$$= \frac{U_{0,1}^2 \sum_{k=1}^{n} \mu_{0,k}^2}{U_{i,1}^2 \sum_{k=1}^{n} \mu_{i,k}^2} \tag{9}$$

$$F_I = \frac{I_0^2}{I_i^2} = \frac{\sum_{k=1}^{n} I_{0,k}^2}{\sum_{k=1}^{n} I_{i,k}^2} = \frac{I_0^2 \sum_{k=1}^{n} a_{0,k}^2}{I_i^2 \sum_{k=1}^{n} a_{i,k}^2} = \frac{I_{0,1}^2 \sum_{k=1}^{n} \varepsilon_{0,k}^2}{I_{i,1}^2 \sum_{k=1}^{n} \varepsilon_{i,k}^2} \tag{10}$$

and, the apparent power transfer function:

$$F_{S_A} = \frac{S_{A,0}^2}{S_{A,i}^2} = \frac{\sum_{k=1}^{n} U_{0,k}^2 \sum_{k=1}^{n} I_{0,k}^2}{\sum_{k=1}^{n} I_{i,k}^2 \sum_{k=1}^{n} I_{i,k}^2} =$$

$$= \frac{U_0^2 I_0^2 \sum_{k=1}^{n} a_{0,k}^2 \sum_{k=1}^{n} b_{0,k}^2}{U_i^2 I_i^2 \sum_{k=1}^{n} a_{i,k}^2 \sum_{k=1}^{n} b_{i,k}^2} = \frac{U_{0,1}^2 I_{0,1}^2 \sum_{k=1}^{n} \varepsilon_{0,k}^2 \sum_{k=1}^{n} \mu_{0,k}^2}{U_{i,1}^2 I_{i,1}^2 \sum_{k=1}^{n} \varepsilon_{i,k}^2 \sum_{k=1}^{n} \mu_{i,k}^2} \tag{11}$$

where: U_o, U_i, I_o, I_i represent the RMS values of output and input tension, respectively current; $S_{A,o}$, $S_{A,i}$ represent the apparent power of output respectively input; $U_{o,1}$, $U_{i,1}$, $I_{o,1}$, $I_{i,1}$ represent the values of fundamental of output and input tension respectively current; $b_{o,k}$, $b_{i,k}$, $\mu_{o,k}$, $\mu_{i,k}$ represent the relative values of harmonics of output and input tension; $a_{o,k}$, $a_{i,k}$, $\varepsilon_{o,k}$, $\varepsilon_{i,k}$ represent the relative values of harmonics of output and input current.

The first order sensitivity of such steady state linear network, related to the relative input values $b_{i,j}$, $\mu_{i,j}$, $a_{i,j}$, $\varepsilon_{i,j}$, can be expressed, with positive values, in the form of [4]:

$$S(F_U, b_{i,j}) = \left| \frac{b_{i,j}}{F_U} \frac{\partial F_U}{\partial b_{i,j}} \right| = \frac{2 b_{i,j}^2}{\sum_{k=1}^{n} b_{i,k}^2} \tag{12}$$

$$S(F_U, \mu_{i,j}) = \left| \frac{\mu_{i,j}}{F_U} \frac{\partial F_U}{\partial \mu_{i,j}} \right| = \frac{2 \mu_{i,j}^2}{\sum_{k=1}^{n} \mu_{i,k}^2} \tag{13}$$

$$S(F_I, a_{i,j}) = \left| \frac{a_{i,j}}{F_I} \frac{\partial F_I}{\partial a_{i,j}} \right| = \frac{2 a_{i,j}^2}{\sum_{k=1}^{n} a_{i,k}^2} \tag{14}$$

$$S(F_I, \varepsilon_{i,j}) = \left| \frac{\varepsilon_{i,j}}{F_I} \frac{\partial F_I}{\partial \varepsilon_{i,j}} \right| = \frac{2 \varepsilon_{i,j}^2}{\sum_{k=1}^{n} \varepsilon_{i,k}^2} \tag{15}$$

$$S(F_{S_A}, a_{i,j}, b_{i,j}) =$$

$$= \left| \frac{a_{i,j}}{F_{S_A}} \frac{\partial F_{S_A}}{\partial a_{i,j}} + \frac{b_{i,j}}{F_{S_A}} \frac{\partial F_{S_A}}{\partial b_{i,j}} \right| =$$

$$= \frac{2}{\sum_{k=1}^{n} a_{i,k}^2 \sum_{k=1}^{n} b_{i,k}^2} \left(a_{i,j}^2 + b_{i,j}^2 \right) \tag{16}$$

$$S(F_{S_A}, \varepsilon_{i,j}, \mu_{i,j}) =$$

$$= \left| \frac{\varepsilon_{i,j}}{F_{S_A}} \frac{\partial F_{S_A}}{\partial \varepsilon_{i,j}} + \frac{\mu_{i,j}}{F_{S_A}} \frac{\partial F_{S_A}}{\partial \mu_{i,j}} \right| =$$

$$= 2 \left(\frac{\varepsilon_{i,j}^2}{\sum_{k=1}^{n} \varepsilon_{i,k}^2} + \frac{\mu_{i,j}^2}{\sum_{k=1}^{n} \mu_{i,k}^2} \right) \tag{17}$$

3. THE INVARIANT RELATIONSHIPS OF SENSITIVITY

Numerous authors show that in the electrical linear and nonlinear networks the sensitivities satisfy the invariant relationships or inequalities, some of which allow deducing the lower and upper bounds of the sensitivity index. Generally speaking, a significant property of linear networks is to existence of sensitivity invariants, which have the form:

$$\sum_{k=1}^{n} S(F, x_k) = \lambda \quad (18)$$

where λ is a constant.

A simple method for deriving sensitivity invariants employs the concept of homogeneity [2]. If a function $F = F(x_1,\dots,x_n)$ is homogenous and of order λ, the following relationship holds:

$$\frac{x_1}{F}\frac{\partial F}{\partial x_1} + \dots + \frac{x_n}{F}\frac{\partial F}{\partial x_n} = \lambda \quad (19)$$

The above-presented relations allow the direct establishment of some invariant sensitivity relationships. For the functions defined by the relations (12)…(15) then homogeneity given by the relative values $b_{i,j}$, $\mu_{i,j}$, $a_{i,j}$, or $\varepsilon_{i,j}$ is of order 2 ($\lambda = 2$) and, for the functions defined by relations (16) and (17) the homogeneity given by the relative values $a_{i,j}$ and $b_{i,j}$, respectively $\mu_{i,j}$, and $\varepsilon_{i,j}$ is of order 4 ($\lambda = 4$), for all the values j=1,…,n. Using the relations (12)…(17) and the condition (7), (8) the sensitivity invariants will be computed as follows [5]:

$$\sum_{j=1}^{n} S(F_U, b_{i,j}) = \sum_{j=1}^{n} \frac{2b_{i,j}^2}{\sum_{k=1}^{n} b_{i,k}^2} = 2 \quad (20)$$

$$\sum_{j=1}^{n} S(F_U, \mu_{i,j}) = \sum_{j=1}^{n} \frac{2\mu_{i,j}^2}{\sum_{k=1}^{n} \mu_{i,k}^2} = 2 \quad (21)$$

and two analogue relations for $a_{i,j}$ and $\varepsilon_{i,j}$.

Similar:

$$\sum_{j=1}^{n} S(F_{S_A}, a_{i,j}, b_{i,j}) = \sum_{j=1}^{n} 2\left(a_{i,j}^2 + b_{i,j}^2\right) = 4 \quad (22)$$

$$\sum_{j=1}^{n} S(F_{S_A}, \varepsilon_{i,j}, \mu_{i,j}) = \sum_{j=1}^{n} 2\left(\frac{\varepsilon_{i,j}^2}{\sum_{k=1}^{n}\varepsilon_{i,k}^2} + \frac{\mu_{i,j}^2}{\sum_{k=1}^{n}\mu_{i,k}^2}\right) = 4 \quad (23)$$

4. EXAMPLE

For a linear steady state networks whose output and input tensions and currents contain the same number of harmonics, we can calculate the sensitivity related to the variation of magnitude of input harmonics, without knowing the circuit structure.

For example, we consider a linear analogue filter RL, presented in figure 1, whose output and input tensions contain the first 3 harmonics. We must to calculate its sensitivity related to the parameters $a_{i,3}$, for the 3-rd harmonic of input current and $b_{i,3}$, for the 3-rd harmonic of input tension, used two procedures:

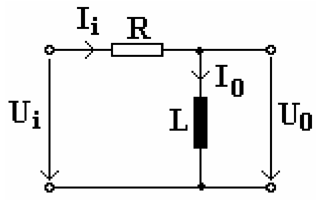

Fig. 1 Linear analogue filter RL

1) If we know, by measurements, for input and output: k-1 relative values a_i, a_o, b_i, b_o, and the RMS values of tension U_i, U_o, and current I_i, I_o, according to relations (9), (12) and (13), we obtain very easy the transfer functions:

$$F_I = \frac{I_0^2}{I_i^2} = \frac{I_0^2 \sum_{k=1}^{3} a_{0,k}^2}{I_i^2 \sum_{k=1}^{3} a_{i,k}^2} \quad (24)$$

$$F_U = \frac{U_0^2}{U_i^2} = \frac{U_0^2 \sum_{k=1}^{3} b_{0,k}^2}{U_i^2 \sum_{k=1}^{3} b_{i,k}^2} \quad (25)$$

Using the relations (14) and (12), the first order sensitivity of such steady state linear network, related to the relative input values $a_{i,3}$

and $b_{i,3}$, will be computed from relations (24) and (25):

$$S(F_I, a_{i,3}) = \left| \frac{a_{i,3}}{F_I} \frac{\partial F_I}{\partial a_{i,j}} \right| = \frac{2 a_{i,3}^2}{\sum_{k=1}^{3} a_{i,k}^2} \quad (26)$$

$$S(F_U, b_{i,3}) = \left| \frac{b_{i,3}}{F_U} \frac{\partial F_U}{\partial b_{i,j}} \right| = \frac{2 b_{i,3}^2}{\sum_{k=1}^{3} b_{i,k}^2} \quad (27)$$

The apparent power transfer function and the first order sensitivity related to the relative input values $a_{i,3}$ and $b_{i,3}$ will be calculated using relations (16) and (17). We obtain:

$$S(F_{S_A}, a_{i,j}, b_{i,j}) =$$

$$= \left| \frac{a_{i,j}}{F_{S_A}} \frac{\partial F_{S_A}}{\partial a_{i,j}} + \frac{b_{i,j}}{F_{S_A}} \frac{\partial F_{S_A}}{\partial b_{i,j}} \right| =$$

$$= 2\left(a_{i,j}^2 + b_{i,j}^2\right) \quad (28)$$

2) If we know: k-1 relative values a_i, a_o, b_i, b_o, the RMS values of tension U_i, U_o, and current I_i, I_o, the parameters and the topology of circuit, we are calculated the tension and current transfer functions:

$$F_I = \frac{I_0^2}{I_i^2} = 1 \quad (29)$$

$$F_U = \frac{U_0^2}{U_i^2} = \frac{\sum_{k=1}^{3} b_{i,k}^2}{\sum_{k=1}^{3} \sqrt{1 + \left(\frac{R}{k\,\omega L}\right)^2}} \quad (30)$$

and, respectively, the sensitivities:

$$S(F_I, a_{i,3}) = \left| \frac{a_{i,j}}{F_I} \frac{\partial F_I}{\partial a_{i,3}} \right| = 1 \quad (31)$$

$$S(F_U, b_{i,3}) = \left| \frac{b_{i,j}}{F_U} \frac{\partial F_U}{\partial b_{i,3}} \right| =$$

$$= \frac{2 b_{i,3}^2}{\sqrt{1 + \left(\frac{R}{3\omega L}\right)^2} \sum_{k=1}^{3} \left(\frac{b_{i,k}^2}{\sqrt{1 + \left(\frac{R}{k\,\omega L}\right)^2}} \right)} \quad (32)$$

Using the parameters of circuit, the apparent power transfer function and the first order sensitivity related to the relative input values $a_{i,3}$ and $b_{i,3}$, are:

$$F_{S_A} = \frac{S_{A,0}^2}{S_{A,i}^2} = \frac{U_0^2 I_0^2 \sum_{k=1}^{3} a_{0,k}^2 \sum_{k=1}^{3} b_{0,k}^2}{U_i^2 I_i^2 \sum_{k=1}^{3} a_{i,k}^2 \sum_{k=1}^{3} b_{i,k}^2} =$$

$$= \frac{\sum_{k=1}^{3} b_{i,k}^2}{\sum_{k=1}^{3} \sqrt{1 + \left(\frac{R}{k\,\omega L}\right)^2}} \quad (33)$$

$$S(F_{S_A}, a_{i,3}, b_{i,3}) =$$

$$= \left| \frac{a_{i,3}}{F_{S_A}} \frac{\partial F_{S_A}}{\partial a_{i,3}} + \frac{b_{i,3}}{F_{S_A}} \frac{\partial F_{S_A}}{\partial b_{i,3}} \right| = \frac{2 a_{i,3}^2}{\sum_{k=1}^{3} a_{i,k}^2} +$$

$$+ \frac{2 b_{i,3}^2}{\sqrt{1 + \left(\frac{R}{3\omega L}\right)^2} \sum_{k=1}^{3} \left(\frac{b_{i,k}^2}{\sqrt{1 + \left(\frac{R}{k\,\omega L}\right)^2}} \right)} \quad (34)$$

It is important to observe that, the sensitivities functions, (31) and (32), respectively (33) and (34), verifies the invariant conditions (20), (21) and (23).

For this circuit we can calculate the numerical values. For example:

1) If we know:

- the values of relative parameters $b_{i,k}$, k = 1,2, and the RMS value of input tension:

$$b_{i,1} = 60\%, \quad b_{i,2} = 5\%, \quad U_i = 100V$$

- the values of relative parameters $b_{i,k}$ k = 1,2, and the RMS value of output tension

$$b_{0,1} = 93{,}8\%, \quad b_{0,2} = 15{,}03\%, \quad U_i = 10{,}51V$$

from relations (7) and (32) results:

$$b_{i,3} = \sqrt{1 - \left(b_{i,1}^2 + b_{i,2}^2\right)} = 65{,}95\%,$$

$$b_{0,3} = \sqrt{1 - \left(b_{0,1}^2 + b_{0,2}^2\right)} = 31{,}3\%,$$

$$S(F_U, b_{i,3}) = 52{,}66\%$$

2) If we know:

- the values of relative parameters $b_{i,k}$, k = 1,2, and the RMS value of input tension:

$b_{i,1} = 60\%, \quad b_{i,2} = 5\%, \quad U_i = 100V$

- the values of relative parameters $b_{i,k}$ k = 1,2, and the RMS of output tension:

$b_{0,1} = 93{,}8\%, \quad b_{0,2} = 15{,}03\%, \quad U_i = 10{,}51V$

- the values of circuit elements: R= 30Ω, L = 10mH, ω = 50Hz, from relations (7) and (32), results:

$$b_{i,3} = \sqrt{1 - \left(b_{i,1}^2 + b_{i,2}^2\right)} = 65{,}95\%,$$

$$b_{0,3} = \sqrt{1 - \left(b_{0,1}^2 + b_{0,2}^2\right)} = 31{,}3\%,$$

$$S(F_U, b_{i,3}) = 52{,}66\%$$

We obtain the same values of sensitivity using the both procedure. The numerical value of sensitivity proves a smaller increase, 52,66% of sensitivity compared to the parameter $b_{i,3}$ = 5,95%.

5. CONCLUSIONS

It is of practical importance to express the sensitivities of the linear steady state networks, especially in the electronic amplifiers and analogue filters, related to the relative values of the input harmonics magnitude, b, μ, a and ε, so that the former should not depend on the circuit parameters. The calculation of the sensitivity of the power generator at different harmonics is of great practical interest in order to obtain a more efficient evaluation of the receptors affected by the harmonics within the network.

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