APPLICATIONS OF MULTIVARIABLE CONTROL TECHNIQUES TO AIRCRAFT GAS TURBINE ENGINES

Constantin ROTARU, Raluca Ioana EDU, Mihai ANDRES-MIHĂILĂ, Pericle Gabriel MATEI

Military Technical Academy, Bucharest, Romania

Abstract: *In this paper are presented two simplified dynamic engine models, based on the general theory of the state variables for linear and nonlinear systems. The constrained optimization problem was formulated using the compact matrix formulas, suitable for the incorporation in MAPLE program solver. The linearized equations were expressed in a matrix form and the engine dynamics was included in terms of variations of the rotational speed following a deflection of the throttle. The linear model of the shaft dynamics for a two-spool jet engine was derived by extending the one-spool model. These models can be used to the entire engine operating envelope, covering a wide range of altitude and Mach number.*

Keywords: *aircraft engine, dynamic model, Laplace transform, control system*

1. INTRODUCTION

The gas turbine engine and its related technologies represent one of the most efficient forms of propulsion and power generation, with applications ranging from land-based power generation, ground-based vehicle propulsion, on-board power and propulsion sources for marine ships, to aircraft propulsion systems.

Design of a gas turbine engine requires the knowledge of multiple academic disciplines including aerodynamics, fluid mechanics, solid mechanics, thermodynamics, chemistry and material sciences (Jaw, 2009).

Controlling such complex machinery requires a thorough understanding of the performance of the engine "system" as a whole.

For some aviation applications, a gas turbine engine must provide a wide range of predictable and repeatable thrust performance over the entire operating envelope of the engine, which can cover the altitude from sea level to tens of thousands meters.

These altitude changes along with variations in flight speed from takeoff to supersonic velocities result in large, simultaneous variations in engine inlet temperature, inlet pressure and exhaust pressure.

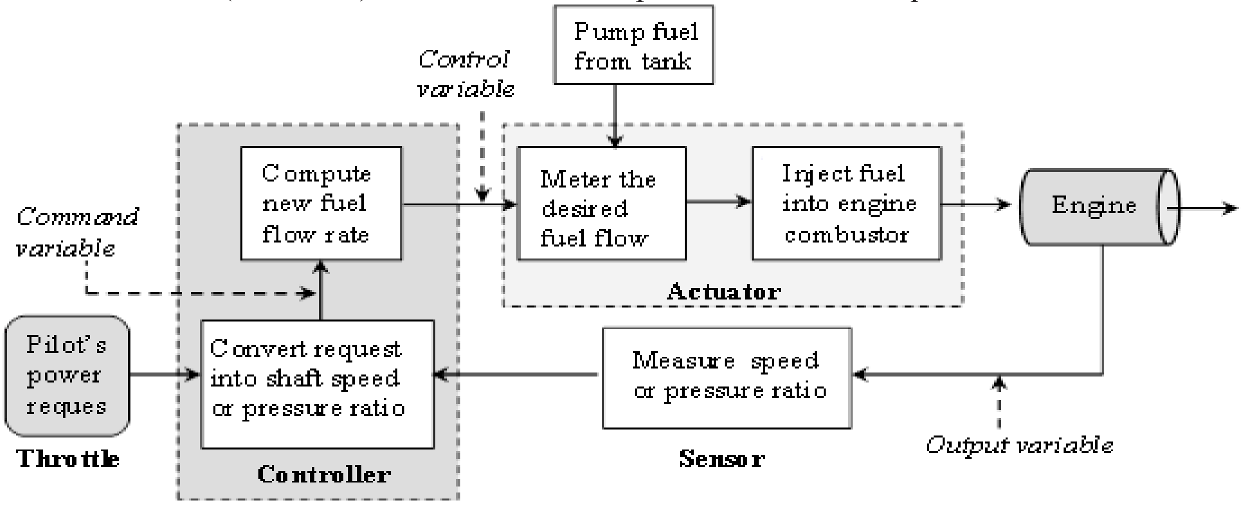

Fig. 1. Functional diagram of a simplified engine control system

These large variations in engine operating conditions and the demand for precise thrust control, coupled with the demand for highly reliable operations, create a significant challenge for the design of the engine control systems (Mattingly, 2006).

Modern gas turbine engine control systems are closed-loop control systems that consist of all four types of control components: controller, sensor, actuator and accessory (Farokhi, 2009).

The simplest engine control system is one that produces desired engine thrust or shaft power by changing the fuel flow (Fig.1).

Because reliable, in-flight engine thrust measurement is not currently practical, the engine shaft rotational speed N or engine pressure ratio (EPR) has been used effectively as an indicator of engine thrust (or power).

Hence, for this simplified control system, the command variable (or the desired output variable) is shaft speed (or engine pressure ratio), the control variable is actuator position, the actuator is fuel metering valve, the output of the metering valve is the fuel flow that is injected in the combustor, the output of the engine is engine power setting variable (shaft speed or engine pressure ratio); furthermore, fuel control accessory components are the fuel tank and the fuel pump and the sensors (Rotaru, 2007).

The resulting compressor pressure ratio and air mass flow rate are plotted on the compressor map once a steady-state has been reached (Fig. 2).

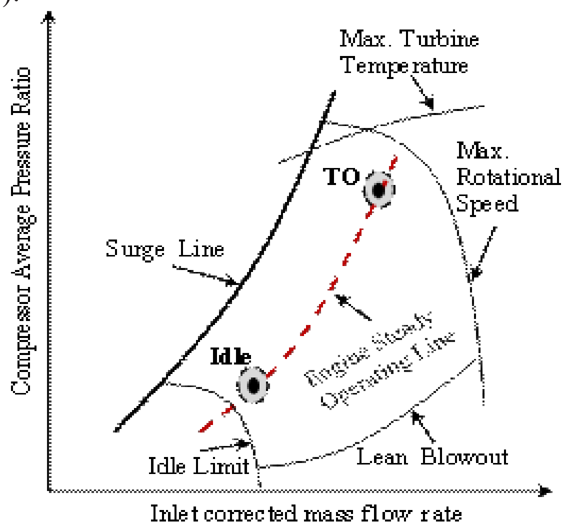

Fig. 2. Engine operating limits on compressor map

Control system complexity can be measured by the number of control variables or by the number of measured variables in the system.

Typically, the number of control variables corresponds directly to the number of actuators and the number of measured variables to the number of sensors.

2. DYNAMIC ENGINE MODELS

Mechanical systems dynamics due to the rotating inertias constitute the most important contribution to the engine transient behavior.

The acceleration of the rotor (consisting of the compressor, turbine and the shaft) based on the principle of Newtonian mechanics is

$$\dot{\omega} = \frac{\Delta Q}{J} \tag{1}$$

where $\dot{\omega}$ is the angular acceleration of the rotor, $\Delta Q = Q_T - Q_C$ represents the difference between the torque produced by the turbine, Q_T, and the torque required by the compressor, Q_C, and J is the mass moment of inertia of the compressor-shaft-turbine body (Fig. 3).

The angular velocity ω is usually substituted by the shaft rotational speed N and the differential torque ΔQ is represented by a function of shaft speed and fuel flow rate W_f.

Substituting these in the torque function, the equation for the shaft rotational speed is expressed as

$$\dot{N} = \frac{f(N, W_f)}{J} \tag{2}$$

Engine dynamics arise from complex, interacting phenomena: gas-flow behavior in the compressor and turbine, shaft inertias and losses, fuel flow transport delay, combustion and the thermal behavior of the engine and its surroundings (Rotaru, 2008).

The linear model of shaft dynamics for one-spool engine, based on the Taylor’s series expansion of the function f at a (steady-state) nominal operating point is (Ronald, 2005)

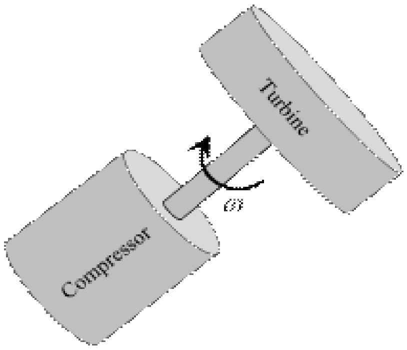

Fig. 3. Engine shaft dynamics

$$\dot{N} = \frac{1}{J}\frac{\partial f}{\partial N}\cdot \Delta N + \frac{1}{J}\frac{\partial f}{\partial W_f}\cdot \Delta W_f \tag{3}$$

The output equation for any engine variable *y* can be expressed as a function of speed and fuel flow as well, so that a small variation of the output variable from its nominal value is expressed as

$$y = \frac{\partial y}{\partial N}\cdot \Delta N + \frac{\partial y}{\partial W_f}\cdot \Delta W_f \tag{4}$$

The transfer function from the input variable fuel flow to the output variable y is expressed as

$$\frac{Y(s)}{W_f(s)} = \frac{b}{s-a} + d \tag{5}$$

where

$$a = \frac{1}{J}\frac{\partial Q}{\partial N};\ b = \frac{1}{J}\frac{\partial Q}{\partial W_f};\ c = \frac{\partial y}{\partial N};\ d = \frac{\partial y}{\partial W_f}$$

For a gas turbine engine, the coefficient a is always less than zero in the control envelope.

Equation (5) represents a first-order lag, which means that the speed response behaves like a lag function after the fuel flow is changed. The linear model of shaft dynamics for a two-spool jet engine can be derived by extending the one-spool model of eq. (3) and eq. (4) with the dynamics of the second shaft.

For a two-spool engine we have

$$\begin{cases} \dot{N}_1 = \dfrac{1}{J_1}\left(\dfrac{\partial Q_1}{\partial N_1}\cdot \Delta N_1 + \dfrac{\partial Q_1}{\partial N_2}\cdot \Delta N_2 + \dfrac{\partial Q_1}{\partial W_f}\cdot \Delta W_f\right) \\ \dot{N}_2 = \dfrac{1}{J_2}\left(\dfrac{\partial Q_2}{\partial N_1}\cdot \Delta N_1 + \dfrac{\partial Q_2}{\partial N_2}\cdot \Delta N_2 + \dfrac{\partial Q_2}{\partial W_f}\cdot \Delta W_f\right) \end{cases}$$

where

$$\begin{cases} \Delta Q_1 = \dot{m}_{t1} c_{p_{t1}}\left(T^*_{4.1} - T^*_4\right) - \dot{m}_{c1} c_{p_{c1}}\left(T^*_{2.1} - T^*_1\right) \\ \Delta Q_2 = \dot{m}_{t1} c_{p_{t2}}\left(T^*_3 - T^*_{4.1}\right) - \dot{m}_{c2} c_{p_{c2}}\left(T^*_2 - T^*_{2.1}\right) \end{cases}$$

The station numbering and the nomenclature used in the above equations are presented at the end of the article.

Similarly, the output equation is given by

$$y = \frac{\partial y}{\partial N_1}\cdot \Delta N_1 + \frac{\partial y}{\partial N_2}\cdot \Delta N_2 + \frac{\partial y}{\partial W_f}\cdot \Delta W_f \tag{6}$$

In the matrix notation, the shaft dynamics for a tow-spool engine are expressed as

$$\begin{bmatrix} \dot{N}_1 \\ \dot{N}_2 \end{bmatrix} = \begin{bmatrix} a_1 & a_2 \\ a_2 & a_2 \end{bmatrix}\begin{bmatrix} N_1 \\ N_2 \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} W_f \tag{7}$$

$$y = \begin{bmatrix} c_1 & c_2 \end{bmatrix}\begin{bmatrix} N_1 \\ N_2 \end{bmatrix} + [d] W_f \tag{8}$$

The frequency-domain representation of two-spool engine dynamics expressed in transfer function form for the output variable y is

$$\frac{Y(s)}{W_f(s)} = C(sI - A)^{-1}B + D = \frac{k(s+z_1)}{(s+r_1)(s+r_2)} \tag{9}$$

where I is the identify matrix. This transfer function represents a second-order dynamic system (Richter, 2012).

The general nonlinear form of the state and the output equations of an engine can de expressed by the following equations

$$\begin{cases} \dot{x}(t) = f[x(t), u(t), t] \\ y(t) = g[x(t), u(t), t] \end{cases} \tag{10}$$

where f and g are nonlinear functions of the state variable, the input variable and time. For gas turbine engine, f and g are smooth enough, within the engine's operating envelope, to have a Taylor-series approximation around the nominal operating condition x_0 and u_0.

3. NUMERICAL RESULTS

Starting from hypothesis that compressor air flow rate, G_a, is equal to the turbine gas flow rate, G_{gT}, applying the energy and mass conservation theorems, one can get

$$\begin{cases} \dfrac{\pi J}{30}\dfrac{dN}{dt} = M_T - M_C \\ \dfrac{T_2^*}{T_1^*} = 1 + \left(\pi_C^{*\frac{\gamma-1}{\gamma}} - 1\right)\dfrac{1}{\eta_C^*} \\ G_a = G_{gT} \\ \dfrac{T_4^*}{T_3^*} = 1 - \left(1 - \dfrac{1}{\pi_C^{*\frac{\gamma'-1}{\gamma'}}}\right)\eta_T^* \\ G_{gT} = G_{noz} \\ W_f H_u \eta_a = c_p G_a\left(T_3^* - T_2^*\right) \end{cases} \tag{11}$$

where N is the rotational speed, J- inertia momentum, G-flow rate for air and gases, $\mathrm{W_f}$ -fuel flow rate, $\mathrm{H_u}$ - low heating value of fuel. For a tow-spool engine the above equations become:

$$\begin{cases} (T_1 \cdot s + \rho_1)\overline{n}_1 - k_{T_{\mathbf{4}}^*}^{(1)}\overline{T}_{\mathbf{4}}^* + k_{p_4^*}^{(1)}\overline{p}_4^* - k_{p_{\mathbf{4}}^*}^{(1)}\overline{p}_{\mathbf{4}}^* + k_{p_{\mathbf{2}}^*}^{(1)}\overline{p}_{\mathbf{2}}^* = 0 \\ (T_2 \cdot s + \rho_2)\overline{n}_2 - k_{T_3^*}^{(2)}\overline{T}_3^* - k_{p_2^*}^{(2)}\overline{p}_2^* + k_{p_{\mathbf{2}}^*}^{(2)}\overline{p}_{\mathbf{2}}^* + k_{p_4^*}^{(2)}\overline{p}_{\mathbf{4}}^* = 0 \\ \overline{T}_2^* - k_{p_2^*}^{(3)}\overline{p}_2^* + k_{p_{\mathbf{2}}^*}^{(3)}\overline{p}_{\mathbf{2}}^* = 0 \\ k_{n_1}^{(4)}\overline{n}_1 - k_{n2}^{(4)}\overline{n}_2 + k_{p_{\mathbf{2}}^*}^{(4)}\overline{p}_{\mathbf{2}}^* - k_{p_2^*}^{(4)}\overline{p}_2^* = 0 \\ k_{p_{\mathbf{2}}^*}^{(5)}\overline{p}_{\mathbf{2}}^* + k_{n2}^{(5)}\overline{n}_2 - k_{p_2^*}^{(5)}\overline{p}_2^* - k_{T_3^*}^{(5)}\overline{T}_3^* = 0 \\ \overline{T}_{\mathbf{4}}^* - \overline{T}_3^* - k_{p_2^*}^{(6)}\overline{p}_2^* - k_{p_4^*}^{(6)}\overline{p}_{\mathbf{4}}^* = 0 \\ \overline{T}_4^* - \overline{T}_{\mathbf{4}}^* - k_{p_4^*}^{(7)}\overline{p}_{\mathbf{4}}^* - k_{p_4^*}^{(7)}\overline{p}_4^* = 0 \\ k_{p_2^*}^{8}\overline{p}_2^* + k_{T_3^*}^{(8)}\overline{T}_3^* - k_{p_4^*}^{(8)}\overline{p}_{\mathbf{4}}^* - k_{T_{\mathbf{4}}^*}^{(8)}\overline{T}_{\mathbf{4}}^* = 0 \\ k_{p_4^*}^{(9)}\overline{p}_{\mathbf{4}}^* + k_{T_{\mathbf{4}}^*}^{(9)}\overline{T}_{\mathbf{4}}^* - k_{p_4^*}^{(9)}\overline{p}_4^* - k_{T_4^*}^{(9)}\overline{T}_4^* = k_{S_a}^{(9)}\overline{S}_a \\ k_{p_2^*}^{(10)}\overline{p}_2^* + k_{n2}^{(10)}\overline{n}_2 + k_{p_{\mathbf{2}}^*}^{(10)}\overline{p}_{\mathbf{2}}^* + k_{T_3^*}^{(10)}\overline{T}_3^* - k_{T_2^*}^{(10)}\overline{T}_2^* = k_{W_f}^{(10)}\overline{W}_f \end{cases}$$

The transfer function from the input variables fuel flow, $\overline{\mathrm{W}}_{\mathrm{f}}(\mathrm{s})$, and the exit nozzle area $\overline{S}_a$, to the output variable, LPC rotational speed $\overline{\mathrm{N}}_1(\mathrm{s})$ and HPC rotational speed $\overline{\mathrm{N}}_2(\mathrm{s})$, for a two-spool engine with

$$G_a = 90\,kg/s;\ T_3^* = 1500\,K;\ \pi_{C1} = 4;\ \pi_{C2} = 7;$$
$$I_1 = 9.9\,kg\cdot m^2;\ I_2 = 11.43\,kg\cdot m^2;$$
$$n_1 = 9000\,rot/\min;\ n_2 = 11000\,rot/\min;$$

are

$$\overline{N}_1(s) = \frac{(1.3\,s + 2.95)\overline{W}_f + (3.65\,s + 6.27)\overline{S}_a}{1.10s^2 + 4.9\ \ s + 5.60}$$

$$\overline{N}_2(s) = \frac{(2.51s + 5.87)\overline{W}_f - 3.32\overline{S}_a}{1.10s^2 + 4.9\ \ s + 5.60}$$

The responses of the rotational speeds $\overline{\mathrm{N}}_1(\mathrm{t}) = \Delta\mathrm{N}_1/\mathrm{N}_1$ and $\overline{\mathrm{N}}_2(\mathrm{t}) = \Delta\mathrm{N}_2/\mathrm{N}_2$ to an impulse input (Dirac function), to a unit step input (Heaviside function) and to a sinusoidal input ($\mathrm{s} = \mathrm{i}\omega$) for $\overline{\mathrm{W}}_{\mathrm{f}}$ and $\overline{\mathrm{F}}_{\mathrm{a}}$ are presented in Figures 4-12.

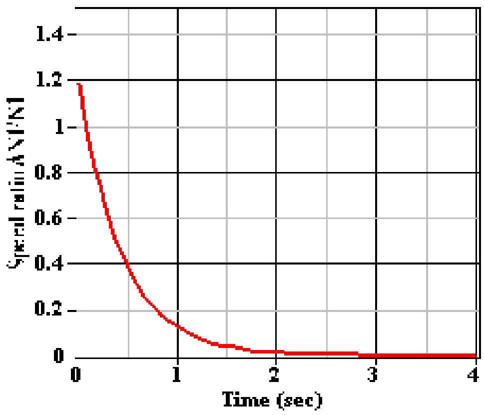

Fig. 4. Impulse response of the LPC rotational speed to the fuel flow rate input

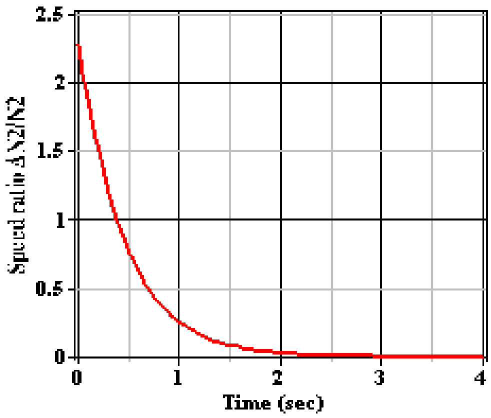

Fig. 5. Impulse response of the HPC rotational speed to the fuel flow rate input

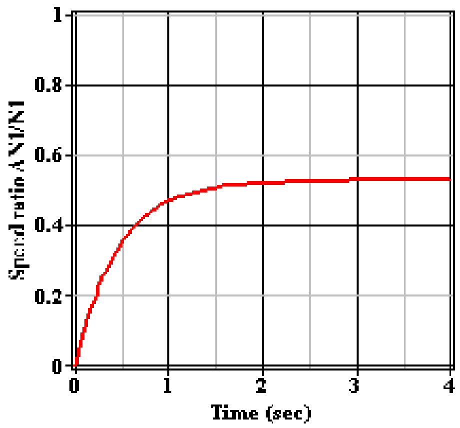

Fig. 6. Step response of the LPC rotational speed to the fuel flow rate input

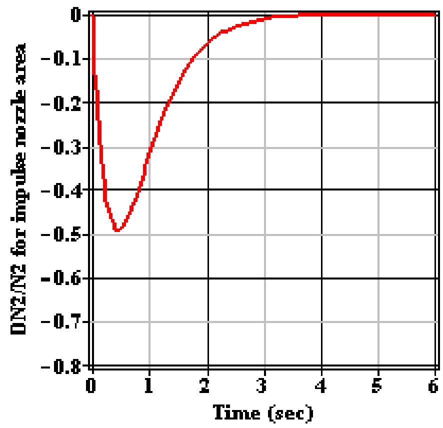

Fig. 9. Impulse response of the HPC rotational speed to the nozzle area input

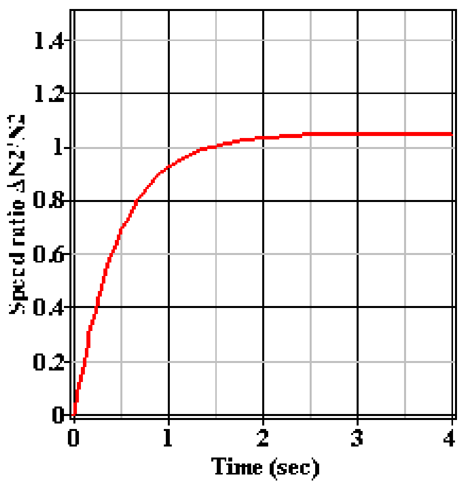

Fig. 7. Step response of the HPC rotational speed to the fuel flow rate input

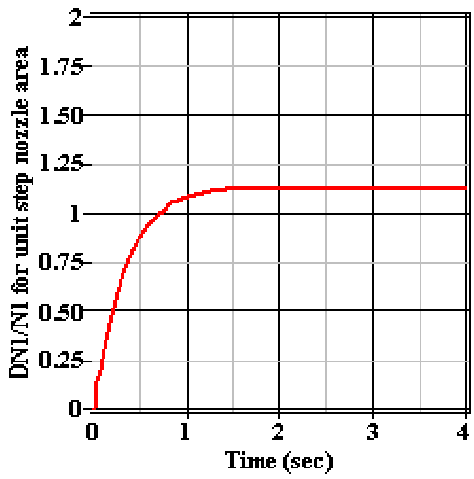

Fig. 10. Step response of the LPC rotational speed to the nozzle area input

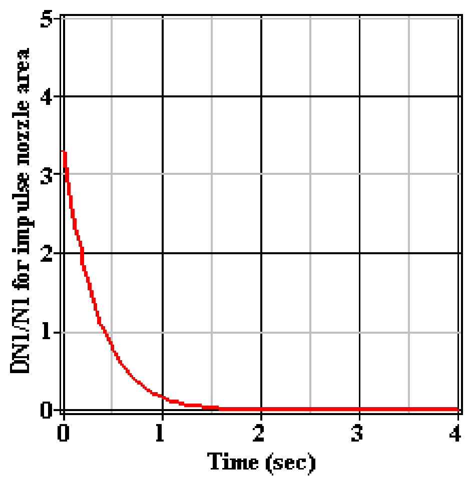

Fig. 8. Impulse response of the LPC rotational speed to the nozzle area input

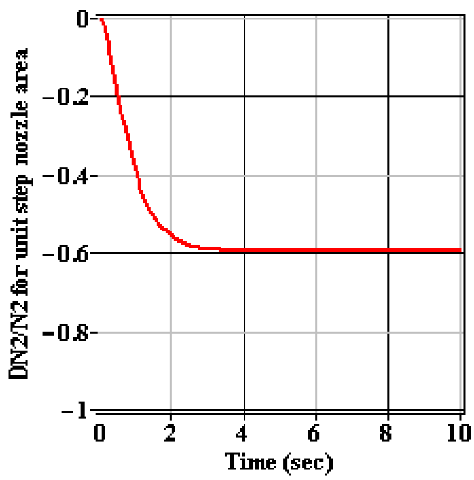

Fig. 11. Step response of the HPC rotational speed to the nozzle area input

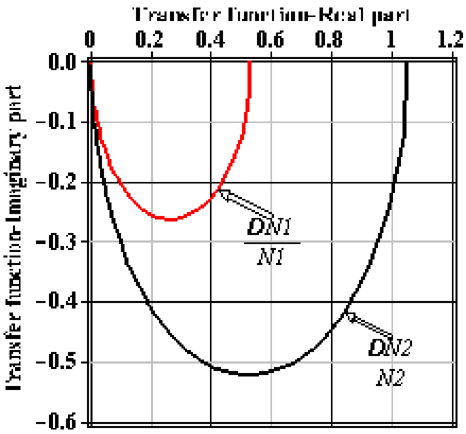

Fig. 12. Frequency response of the LPC and HPC rotational speed

4. CONCLUSIONS

The classical linear compensation is adequate only to govern the engine close to a fixed operating point, as defined by the current inlet conditions and desired thrust set point.

Aside from nonlinearity and parametric changes in the engine, critical variables must be maintained within safety ranges.

Linear compensation is the basic building block of standard aircraft engine control system.

Parametric changes and nonlinearity are addressed with gain-scheduled linear compensators while limit protection logic schemes are used to override the active linear regulator when a critical variable approaches its safety limit.

Even with constant control gains, limit relaxation is reflected in faster responses, and conversely, the main output response will become slower if limits are made more restrictive.

There are two ways of obtaining faster thrust response: redesigning the regulators for larger closed-loop bandwidths and relaxing the protective limits on variables which tend to peak as thrust response is made faster.

Among the variables displaying such peaking are turbine outlet temperature, which peaks during acceleration, stall margin, which tends to undershoot during acceleration and combustor pressure, which tends to undershoot during deceleration.

NOMENCLATURE

The naming convention for the symbols representing engine models and control laws follows the convention defined as: T^* - total temperature [K]; p^* - total pressure $\left[N/m^2\right]$; π - pressure ratio; G and W_f - mass flow rate $\left[kg/s\right]$; σ_a - combustion chamber total pressure loss coefficient; γ - ratio of specific heat; $\gamma = 1.4$; $\gamma' = 1.3$; c_p - specific heat at constant pressure $\left[J/(kg \cdot K)\right]$; HPC – high pressure compressor; LPC – low pressure compressor; $x_i = (x_i)_0 + \Delta x_i$.

Subscripts: c – compressor; t – turbine; 1-5 engine cross section number; 2.1 – HPC inlet section; 4.1 – LP inlet turbine.

BIBLIOGRAPHY

1. Farokhi, S. (2009), "*Aircraft Propulsion*", John Wiley & Sons Inc., USA.
2. Jaw, L. C., Mattingly, J. D. (2009), "*Aircraft Engine Controls – Design, System Analysis and Health Monitoring*", AIAA, Reston Virginia, USA.
3. Mattingly, J. D. (2006), "*Elements of Propulsion. Gas Turbines and Rockets*", AIAA, Reston Virginia, USA.
4. Richter, H. (2012), "*Advanced Control of Turbofan Engines", Springer Science, USA.*
5. Ronald, D. F. (2005), "*Fundamentals of Jet Propulsion with Applications*", Cambridge University Press.
6. Rotaru, C., Safta, D., Barbu, C. (2007), "*Aspects regarding internal flow in combustion chamber of turbojet engines*", Proceedings of the 5th IASME/WSEAS International Conference on Fluid Dynamics and Aerodynamics, Greece.
7. Rotaru, C., Arghiropol, A., Barbu, C. (2008), "*Some aspects regarding possible improvements in the performances of the aircraft engines*", Proceedings of the 6th IASME/WSEAS International Conference on Fluid Dynamics and Aerodynamics, Greece.