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MECHANICAL PROPERTIES OF PLAIN WEAVE FABRICS

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Abstract: *The aim of this work is to develop accurate finite element models of plain weave fabrics to determine their mechanical properties. The geometric models needed for finite element discretization of the plain weave fabrics are developed for glass-epoxy plain-weave reinforced laminate for which experimental data is available in the literature. These include single lamina composites from three sources, as well as laminates in iso-phase and out-of-phase configurations. The procedures to determine the elastic moduli using iso-strain, and classical lamination theory are presented.*

Keywords: *composite materials, fiber, matrix*

1. Theoretical aspects

Textile preforming plays an important role in composite technology providing glass, aramid, carbon, and hybrid fabrics that are widely used as reinforcing materials. The main advantages of woven composites are their cost efficiency and high processability, particularly, in lay-up manufacturing of largescale structures. However, on the other hand, processing of fibers and their bending in the process of weaving results in substantial reduction of material strength and stiffness. As can be seen in Fig. 1, where a typical woven structure is shown the warp (lengthwise) and fill (crosswise) yarns forming the fabric make angle α with the plane of the fabric layer.

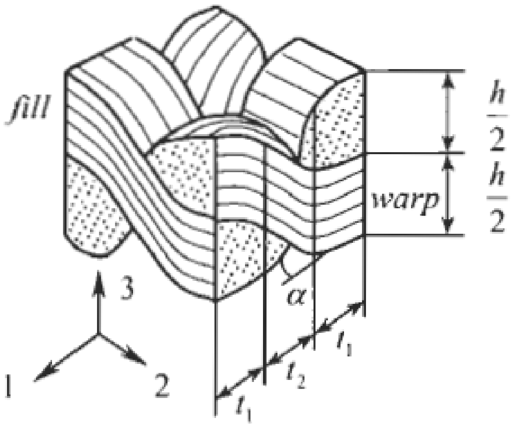

Fig. 1. Unit cell of a fabric structure.

To demonstrate how this angle influences material stiffness, consider tension of the structure shown in Fig. 1 in the warp direction. Apparent modulus of elasticity can be expressed as

$$E_a A_a = E_f A_f + E_w A_w \quad (1)$$

where $A_a = h(2t_1 + t_2)$ is the apparent cross-sectional area and

$$A_f = \frac{h}{2}(2t_1 + t_2),$$

$$A_w = \frac{h}{4}(4t_1 + t_2)$$

are the areas of the fill and warp yarns in the cross section. Substitution into Eq. (1) yields

$$E_a = \frac{1}{2}\left[E_f + \frac{E_w(4t_1 + t_2)}{4(2t_1 + t_2)}\right].$$

Because the fibers of the fill yarns are orthogonal to the loading direction, we can take $E_f = E_2$ where E_2 is the transverse modulus of a unidirectional composite.

Compliance of the warp yam can be decomposed into two parts corresponding to t_1 and t_2 in Fig. 1, i.e.,

$$\frac{2t_1 + t_2}{E_w} = \frac{2t_1}{E_1} + \frac{t_2}{E_\alpha},$$

where, E_1 is the longitudinal modulus of a unidirectional composite, while is

$$\frac{1}{E_\alpha} = \frac{\cos^4\alpha}{E_1} + \frac{\sin^4\alpha}{E_2} + \left(\frac{1}{G_{12}} - \frac{2\nu_{21}}{E_1}\right)\sin^2\alpha\cos^2\alpha \quad (2)$$

The final result is as follows:

$$E_a = \frac{E_2}{2} + \frac{E_1(4t_1 + t_2)}{4\left\{2t_1 + t_2\left[\cos^4\alpha + \frac{E_1}{E_2}\sin^4\alpha + \left(\frac{E_1}{G_{12}} - 2\nu_{21}\right)\sin^2\alpha\cos^2\alpha\right]\right\}} \quad (3)$$

For example, consider a glass fabric with the following parameters: $a = 12°$, $t_2 = 2t_1$. Taking elastic constants of a unidirectional material from Table 3.5 we get for the fabric composite $E_a = 23.5\,\text{GPa}$. For comparison, a cross-ply [0°/90°] laminate made of the same material has $E = 36.5\,\text{GPa}$. Thus, the modulus of a woven structure is by 37% less than the modulus of the same material but reinforced with straight fibers. Typical mechanical characteristics of fabric composites are listed in Table 1.

Table 1 - Typical properties of fabric composites.

Property	Glass fabric-epoxy	Aramid fabric-epoxy	Carbon fabric-epoxy
Fiber volume fraction	0.43	0.46	0.45
Density (g/cm3)	1.85	1 .25	1 .40
Longitudinal modulus (GPa)	26	34	70
Transverse modulus (GPa)	22	34	70
Shear modulus (GPa)	7.2	5.6	5.8
Poisson's ratio	0.13	0.15	0.09
Longitudinal tensile strength (MPa)	400	600	860
Longitudinal compressive strength (MPa)	350	150	560
Transverse tensile strength (MPa)	380	500	850
Transverse compressive strength (MPa)	280	150	560
In-plane shear strength (MPa)	45	44	150

Stiffness and strength of fabric composites depend not only on the yarns and matrix properties, but on material structural parameters, i.e., on fabric count and weave, as well. The fabric count specifies the number of warp and fill yarns per inch (25.4 mm), while the weave determines how the warp and the fill yarns are interlaced. Typical weave patterns are shown in Fig. 4.81 and include plain, twill, and satin. In the plain weave (see Fig. 2a) which is the most common and the oldest, the warp yarn is repeatedly woven over the fill yarn and under the next fill yarn. In the twill weave, the warp yarn passes over and under two (as in Fig. 2b) or more fill yarns in a regular way. A structure with one warp yarn passing over four and under one fill yarn is referred to as a five harness satin weave (Fig. 2c).

Being formed from one and the same type of yarns plain, twill, and satin weaves provide approximately the same strength and stiffness of the fabric in the warp and the fill directions. Typical stress-strain diagrams for a fiberglass fabric composite of such a type are presented in Fig. 3. As can be seen, material demonstrates relatively low stiffness and strength under tension at the angle of 45° with respect to the warp or fill directions. To improve these properties, multiaxial woven fabrics, one of which is shown in Fig. 2d, can be used.

Fabric materials whose properties are more close to those of unidirectional composites are made by weaving a great number of larger yarns in longitudinal direction and fewer and smaller yarns in the orthogonal direction. Such weave is called unidirectional. It provides materials with high stiffness and strength in one direction, which is specific for unidirectional composites and high processability typical for fabric composites.

Although microstructural models of the type shown in Fig. 1 and leading to equations similar to Eq. 3 have been developed to predict stiffness and even strength characteristics of fabric composites (e.g., Skudra et al., 1989), for practical design and analysis, these characteristics are usually determined by experimental methods. Elastic constants entering constitutive equations written in the principal material coordinates are found testing strips cut out of fabric composite plates at different angles with respect to the orthotropy axes. The 0° and 90° specimens are used to determine moduli of elasticity El, E2 and Poisson's ratios v12, v21 (or parameters of nonlinear stress-strain diagrams), while the in-plane shear stiffness can be obtained with the aid of off-axis tension.

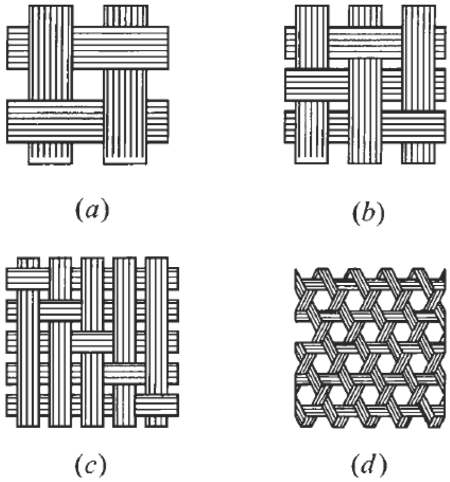

Fig. 2. Plain (a), twill (b), satin (c), and triaxial (d) woven fabrics.

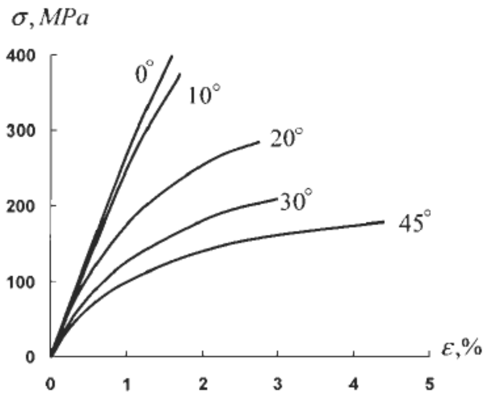

Fig. 4. Stress-strain curves for fiber glass fabric composite loaded in tension at different angles with respect to the warp direction

Assuming that there is no shear-extension coupling $(\eta_{x,xy} = 0)$ we can write the following equations:

$$\frac{1}{E_x} = \frac{1+\nu_{21}}{E_1}\cos^4\phi + \frac{1+\nu_{12}}{E_2}\sin^4\phi - \frac{\nu_{21}}{E_1} + \frac{1}{G_{12}}\sin^2\phi\cos^2\phi \quad (4a)$$

$$\frac{\nu_{yx}}{E_x} = \frac{\nu_{21}}{E_1} - $$
$$-\left(\frac{1+\nu_{21}}{E_1} + \frac{1+\nu_{12}}{E_2} - \frac{1}{G_{12}}\right)\sin^2\phi\cos^2\phi \tag{4b}$$

$$\frac{1+\nu_{21}}{E_1}\cos^2\phi - \frac{1+\nu_{12}}{E_2}\sin^2\phi - $$
$$-\frac{1}{2G_{12}}\cos 2\phi = 0 \tag{4c}$$

Summing up the first two of these equations we get

$$\frac{1+\nu_{yx}}{E_x} = \frac{1+\nu_{21}}{E_1}\cos^2\phi - \frac{1+\nu_{12}}{E_2}\sin^2\phi + $$
$$+\frac{2}{G_{12}}\sin^2\phi\cos^2\phi$$

Using the third equation we arrive at the remarkable result

$$G_{12} = \frac{E_x}{2\left(1+\nu_{yx}\right)} \tag{5}$$

similar to the corresponding formula for isotropic materials. It should be emphasized that Eq. (5) is valid for off-axis tension in the x-direction making some special angle ϕ with the principal material axis 1. Another form of this expression follows from the last equation of Eqs. (4) and (5), i.e.,

$$\sin^2\phi = \frac{\frac{1+\nu_{yx}}{E_x} - \frac{1+\nu_{21}}{E_1}}{2\frac{1+\nu_{yx}}{E_x} - \frac{1+\nu_{21}}{E_1} - \frac{1+\nu_{12}}{E_2}} \tag{6}$$

For fabric composites whose stiffness in the warp and the fill directions is the same ($E_1 = E_2$), Eq. (6) yields $\phi = 45°$.

2. FINITE ELEMENT MODELING

A representative volume element (RVE) encompassing one full wavelength in the warp and fill directions (two pitches, or 2a, which is twice the length shown in Fig. 5), exhibits geometric and material periodicity. Therefore, it can be used to analyze the composite by imposing periodicity conditions on its boundary.

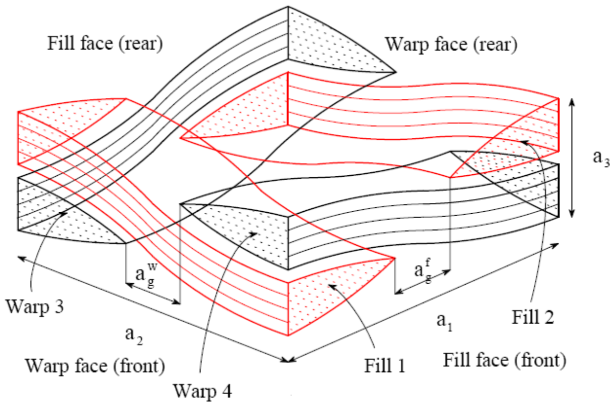

Figure 5. Schematic representation of the Fabric geometry

The RVE dimensions are $a_1 = a_2 = 870\ \mu m$ and $a_3 = 190\ \mu m$. The fill and warp fibers are the same and for the geometrical construction it used seven sections (figure 6).

The 3D geometric models are meshed using 8 node solid brick under the parametric mesh option in COSMOSM (SOLID element) – Figure 8. Each node has 3 degrees of freedom, u_x, u_y and u_z.

For computation of the axial moduli and Poisson's ratios, symmetry conditions are imposed on one warp face (perpendicular to the warp tows) and on one fill face (perpendicular to the fill tows). Coupling conditions (CP) are used to keep the remaining warp and fill faces plane as they deform under load. This is necessary to avoid violating the symmetry conditions on those faces.

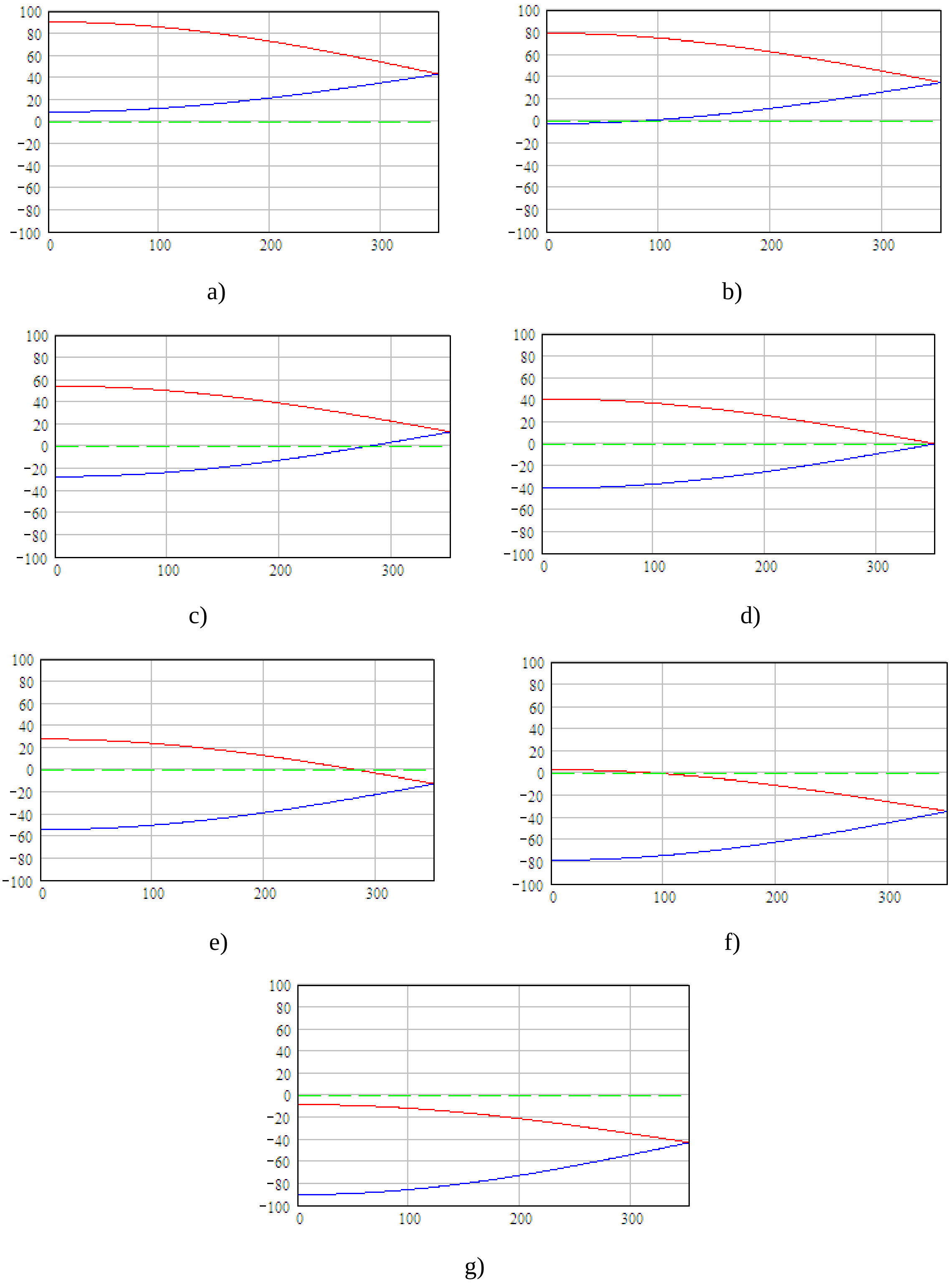

Figure 6. The position of the section in respect whit middle plane of RVE

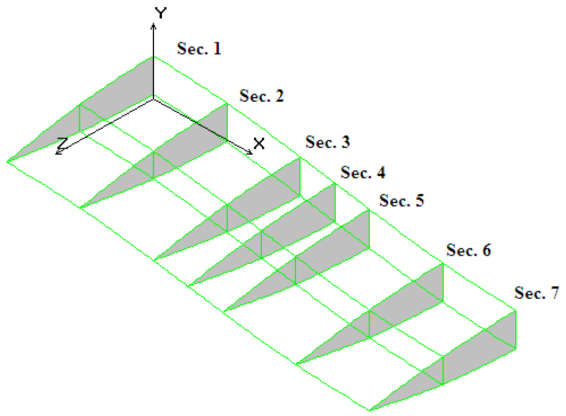

Figure 7. 3D representation of a fiber

Elastic moduli can be computed by imposing either a uniform average stress (isostress) or uniform average strain (iso-strain). For computation of the axial moduli and Poisson's ratios, iso-stress conditions are applied by imposing a concentrated force to one face at a time, while displacements in the direction of the force are coupled so that the applied force effectively translates into an average stress. All remaining faces are let free but coupled so that their deformation remains plane. This results in a set of displacements that, by virtue of the coupling conditions, results in a set of average strains.

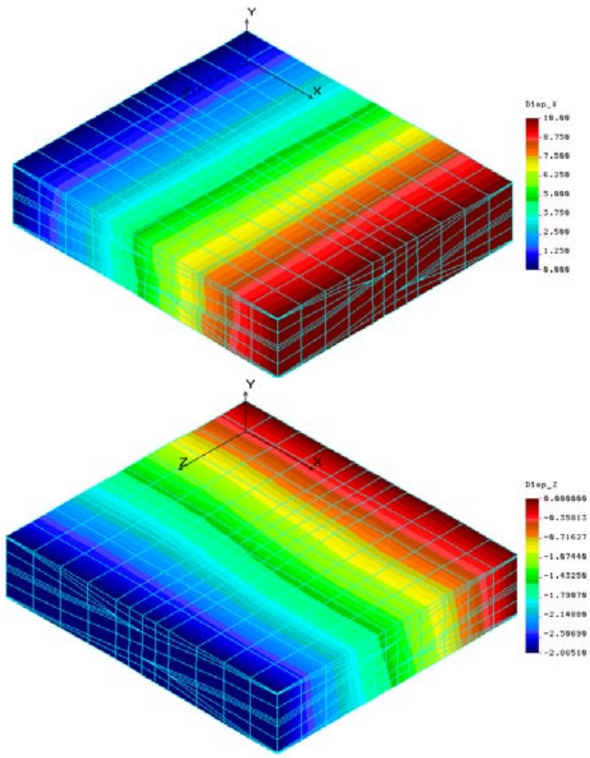

Figure 8. Ux and Uz displacent

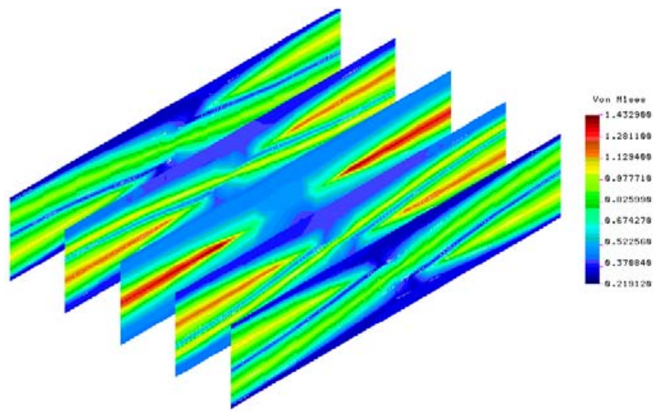

Figure 9 Von Misses stress

3. Numerical determiantion

The cross sectional areas of RVE are

$$A_1 = a_2 \cdot a_3 \qquad A_1 = 0{,}165\left(mm^2\right)$$
$$A_2 = a_1 \cdot a_2 \qquad A_2 = 0{,}757\left(mm^2\right) \quad (7)$$
$$A_3 = a_2 \cdot a_3 \qquad A_3 = 0{,}165\left(mm^2\right)$$

Imposing an $\Delta x = 8,7\mu m$ displacement on fill face

$$\varepsilon_x = \frac{\Delta x}{a_1} = \frac{8,7\mu m}{870\mu m} = 0,01 \qquad (8)$$

The resulted reaction force on the opposite face is

$$R_x = 18{,}434\left(N\right) \qquad (9)$$

and the average stress

$$\sigma_x = \frac{R_x}{A_1} = 111{,}719 \left(\frac{N}{mm^2}\right). \qquad (10)$$

The mesasured contractions on the RVE are

$$\begin{cases} \Delta y = -0{,}508 & (\mu m) \\ \Delta z = -1{,}256 & (\mu m) \end{cases} \qquad (11)$$

and the average strains are

$$\begin{cases} \varepsilon_y = \dfrac{\Delta y}{a_3} = -0{,}00683 \\ \varepsilon_z = \dfrac{\Delta z}{a_2} = -0{,}00142 \end{cases} \quad (12)$$

For the ortotrophic material case it results a 3 equation system with 9 unknowns:

$$\begin{Bmatrix} 111{,}719 \\ 0 \\ 0 \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} \cdot \begin{Bmatrix} 0{,}01000 \\ -0{,}00683 \\ -0{,}00142 \end{Bmatrix} \quad (13)$$

By repeating the procedure for the other two principal directions the system can be solved, resulting the mechanical properties of the RVE.

The shear moduli are calculated by imposing two displacements u_1 and u_2 on fill and warp faces, and the average shear strain is:

$$\gamma_{xy}^0 = \frac{u_1(x, d_2, z)}{d_2} + \frac{u_2(d_1, y, z)}{d_1} \quad (14)$$

the reaction force f_1 and f_2 on the two faces leads to average shear stress:

$$\tau_{xy}^0 = \frac{f_1(y=0)}{a_1 a_3} = \frac{f_2(x=0)}{a_2 a_3} \quad (15)$$

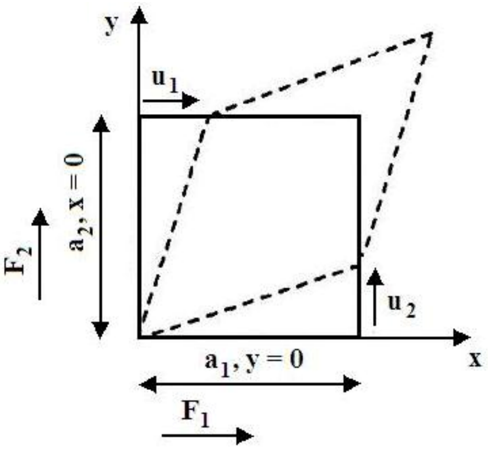

Figure 10. The condition used for shear moduli determination

For

$$\begin{cases} d_1 = a_2 = 870(\mu m) \\ d_2 = a_3 = 190(\mu m), \\ d_3 = a_1 = 870(\mu m) \end{cases} \quad (16)$$

the average shear strain:

$$\gamma_{xy}^0 = \frac{u_1}{d_2} + \frac{u_2}{d_1} = 0{,}048 \quad (17)$$

For reaction forces:

$$\begin{cases} f_1 = F_{yx} = 85{,}587(N) \\ f_2 = F_{xy} = 18{,}691(N) \end{cases} \quad (18)$$

the average shear stress:

$$\tau_{xy} = \frac{\tau_{yx} + \tau_{xy}}{2} = 113{,}075\left(\frac{N}{mm^2}\right) \quad (19)$$

and

$C_{66} = 0{,}048/113{,}075$.

The elastic properties for the RVE are

$$\begin{array}{lll} E_x = 11{,}172(GPa) & G_{yz} = 2{,}338(GPa) & \upsilon_{yz} = 0{,}188 \\ E_y = 7{,}8410(GPa) & G_{xz} = 4{,}391(GPa) & \upsilon_{xz} = 0{,}143 \\ E_z = 11{,}165(GPa) & G_{xy} = 2{,}357(GPa) & \upsilon_{xy} = 0{,}268 \end{array}$$

4. Conclusions

The geometric model is based on microphotograph measurements that are translated into a solid model and a FEM model using commercial software. The elastic moduli of the plain weave fabric-reinforced laminates are obtained using finite element analysis. The values predicted by the FEM models compare favorably with the experimental values. The model is simple; as it is based on microphotograph measurements and the stiffness values of a unidirectional composite that can be obtained from standard tests.

Bibiograpy

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