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ASPECTS REGARDING THE UNSTEADY AERODYNAMICS AND ITS APPLICATION TO HELICOPTER ROTOR BLADE

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Abstract: *The principal focus of the presence study is to describe the key physical features and techniques for modeling the unsteady aerodynamic effects found on helicopter rotor blade operating under nominally attached flow conditions away from stall. The unsteady effects were considered as phase differences between the forcing function and the aerodynamic response, being functions of the reduced frequency, the Mach number and the mode forcing. For a helicopter rotor, the reduced frequency at any blade element can't be exactly calculated but a first order approximation for the reduced frequency gives useful information about the degree of unsteadiness. The sources of unsteady effects were decomposed into perturbations to the local angle of attack and velocity field.*

Keywords: *aerodynamics, unsteady models, helicopter*

1. INTRODUCTION

The classical unsteady aerodynamic theories describing the observed behavior have formed the basis for many types of rotor analysis. The tools for the analysis of 2-D, incompressible, unsteady aerodynamic problems were extended to compressible flows, being a basis for developing linearized unsteady aerodynamic models applicable to compressible flows. But, while the classical theories assume linearity in the airloads, the assumption of linearity can probably be justified for many of the problems encountered on the rotor, in practice. The advent of nonlinear methods based on CFD solutions to the Euler and Navier-Stokes equations has provided new results that justify and define the limits of the linear models and may give guidance in developing improved and more practical unsteady aerodynamic models for future use in helicopter rotor blade airloads prediction, aeroelastic analysis and rotor design. At the blade element level, the various sources of unsteady effects can be decomposed into perturbations to the local angle of attack and velocity field. At low angle of attack with fully attached flow, the various sources of unsteady effects manifest as moderate amplitude and phase variations relative to the quasi-steady airloads. At higher angles of attack when time-dependent flow separation from the airfoil may be involved, a phenomenon characterized by large overshoots in the values of the lift, drag and pitching moment relative to the quasi-steady stall values, may occur.

One important parameter used in the description of unsteady aerodynamics und unsteady airfoil behavior is the reduced frequency, k, defined as $k = \omega \cdot c/(2V)$, where ω is the angular frequency, *c* is the chord of the airfoil and V is the flow velocity. According to the dimensional analysis, the resultant force, F, on the airfoil chord *c* can be written in functional form as $F/(\rho V^2 c^2) = f(Re, M, k)$. For $k = 0$ the flow is steady and for $0 \le k \le 0.05$ the flow can be considered quasi-steady, that is, unsteady effects are generally small. Flows with characteristic reduced frequencies above of 0.05 are considered unsteady.

For a helicopter rotor in forward flight (fig. 1), the reduced frequency at any blade element can't be exactly calculated, but a first order approximation for k, can give useful information about the degree of unsteadiness.

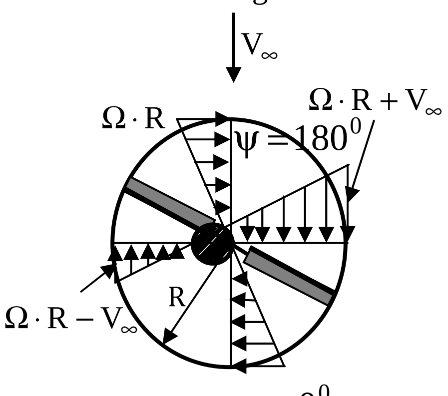

Fig. 1 Main Rotor

The approach to modeling of unsteady aerodynamic effects through an extension of steady, 2-D thin airfoil theory gives a good level of analysis of the problem and provides considerable insight into the physics responsible for the underlying unsteady behavior. The Laplace's equation for incompressible flow is eliptic, therefore the unsteady aerodynamic theories cannot be obtained in a corresponding analytical form.

2. THE AIRLOADS ON AN OSCILLATING AIRFOIL

The oscillatory motion of the airfoil can be decomposed into contributions associated with angle of attack which is equivalent to a pure plunging motion (fig. 2) and contributions associated with pitching (fig. 3).

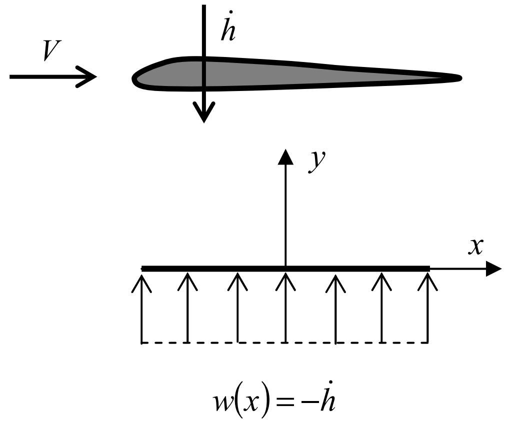

Fig. 2 Plunge Velocity

A plunge velocity $\dot{h}$ produces a uniform velocity perturbation w, that is normal to the chord, $w(x) = -\dot{h}$ and the pitch-rate term produces a linear variation in normal perturbation velocity.

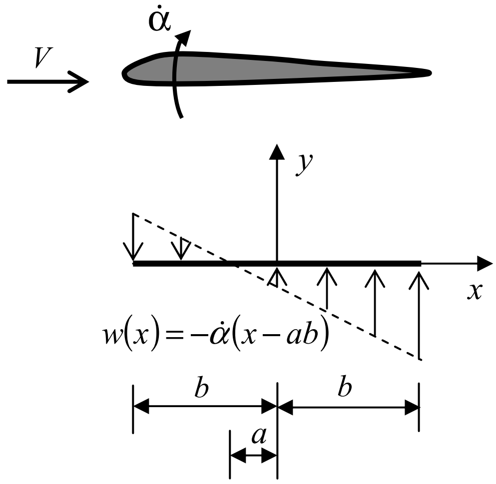

Fig. 3 Pitch Rate

For a pitch rate imposed about an axis at "a" semi-chords from the mid-chord, then $w(x) = -\dot{\alpha}(x - a \cdot b)$, so that the induced chamber is a parabolic arc.

The problem of finding the airloads on an oscillating airfoil was solved by Theodorsen, who gave a solution to the unsteady airloads on a 2-D harmonically oscillated airfoil in inviscid, incompressible flow, with the assumption of small disturbances. Both the airfoil and its shed wake were represented by a vortex sheet with the shed wake extending as a planar surface from the trailing edge downstream to infinity. The assumption of planar wake is justified if the angle of attack disturbances remain relatively small. As with the standard quasi-steady thin airfoil theory, the bound vorticity, γ_b, can sustain a pressure difference and, therefore, a lift force. The wake vorticity, γ_w, must be force free with zero net pressure jump over the sheet. According to the Theodorsen's theory, the solution for the loading γ_b on the airfoil surface under harmonic forcing conditions is obtained from integral equation

$$w(x,t)=\frac{1}{2\pi}\int_0^c \frac{\gamma_b(x,t)}{x-x_0}dx+\frac{1}{2\pi}\int_c^\infty \frac{\gamma_w(x,t)}{x-x_0}dx \quad (1)$$

where w is the downwash on the airfoil surface. At the trailing edge, $\gamma_b(c,t)=0$, and the airfoil circulation $\Gamma(t)$ is given by

$$\Gamma(t)=\int_0^c \gamma_b(x,t)dx \quad (2)$$

So long as the circulation about the airfoil is changing with respect to time, the circulation is continuously shed into the wake and will continuously affect the aerodynamic loads on the airfoil. For a general motion, where an airfoil of chord $c=2b$ is undergoing a combination of pitching $(\alpha,\dot{\alpha})$ and plunging (h) motion in a flow of steady velocity V, Theodorsen's solution for the lift coefficient and pitching moment coefficient corresponding to mid-chord, $M_{1/2}$ are

$$\begin{cases} c_l=\pi b\left[\frac{\ddot{h}}{V^2}+\frac{\dot{\alpha}}{V}-\frac{b}{V^2}a\ddot{\alpha}\right]+ \\ +2\pi\left[\frac{\dot{h}}{V}+\alpha+\frac{b\dot{\alpha}}{V}\left(\frac{1}{2}-a\right)\right]C(k) \\ c_{m1/2}=\frac{\pi}{2}\left[\frac{ba\ddot{h}}{V^2}-\frac{b^2}{V^2}\left(\frac{1}{8}+a^2\right)\ddot{\alpha}\right]+ \\ +\pi\left(a+\frac{1}{2}\right)\left[\frac{\dot{h}}{V}+\alpha+b\left(\frac{1}{2}-a\right)\frac{\dot{\alpha}}{V}\right]C(k)- \\ -\frac{\pi}{2}\left[\left(\frac{1}{2}-a\right)\frac{b\dot{\alpha}}{V}\right] \end{cases}$$

where a is the pitch axis location relative to the mid-chord of the airfoil, measured in terms of semi-chord and $C(k)=F(k)+iG(k)$ is the complex transfer function (known as Theodorsen's function) which accounts the effects of the shed wake on the unsteady airloads.

$$\begin{aligned} C(k)&=\frac{H_1^{(2)}(k)}{H_1^{(2)}(k)+i\cdot H_0^{(2)}(k)}= \\ &=\frac{J_1(J_1+J_0)+Y_1(Y_1-J_0)}{(J_1+Y_0)^2+(Y_0-J_1)^2}+ \\ &+i\frac{Y_1Y_0+J_1J_0}{(J_1+Y_0)^2+(Y_0-J_1)^2} \end{aligned} \quad (3)$$

with J_0, J_1, Y_0, Y_1 being Bessel functions of the first and second kind, respectively (fig. 4).

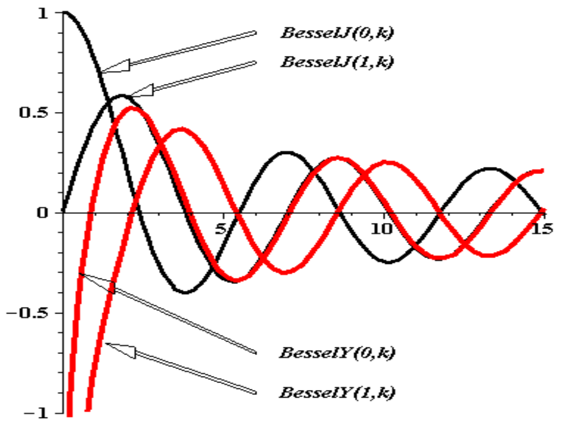

Fig. 4 Bessel Functions

The Hankel functions in above expression are:

$$\begin{cases} H_0^{(2)} = J_0 - i \cdot Y_0 \\ H_1^{(2)} = J_1 - i \cdot Y_1 \end{cases} \qquad (4)$$

The real and imaginary parts of $C(k)$ function are plotted in fig. 5.

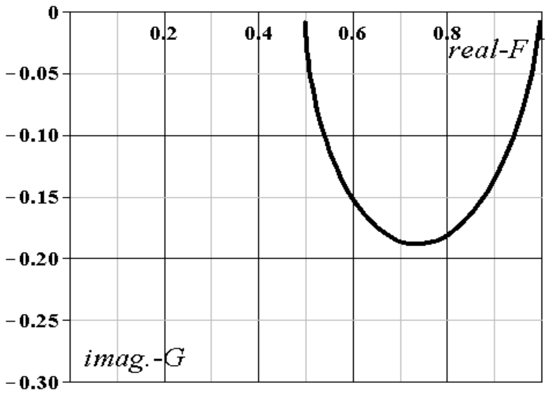

Fig. 5 Theodorsen's Function

It could be appreciated that $C(k)$ function serves to introduce an amplitude reduction and phase lag effect on the circulatory part of the lift response compared to the result obtained under quasi-steady conditions.

This effect can be seen if a pure oscillatory variation in angle of attack is considered, that is, $\alpha = \bar{\alpha}e^{i\omega t}$, so the circulatory part of the airfoil lift coefficient is given by

$$\alpha = 2\pi\bar{\alpha}C(k) = 2\pi\bar{\alpha}[F(k) + iG(k)] \qquad (5)$$

For $k = 0$, the steady-state lift behavior is obtained, that is c_l is linearly proportional to α. As k is increased, the lift plots develop into hysteresis loops and these loops rotate such that the amplitude of the lift response (half of the peak-to-peak value) decreases with increasing reduced frequency. These loops are circumvented in a counterclockwise direction such that the lift is lower than the steady value when α is decreasing with time (i.e., there is a phase lag). For infinite reduced frequency the circulatory part of the lift amplitude is half that at $k = 0$ and there is no phase lag angle.

Pure angle of attack oscillations

For a harmonic variation in α, that is $\alpha = \bar{\alpha}e^{i\omega t}$, the lift is

$$L = 2\pi\rho V^2 b\left[C(k) + \frac{1}{2}i\frac{\omega b}{V}\right]\bar{\alpha}e^{i\omega t} \qquad (6)$$

or, in terms of the lift coefficient, the results is

$$c_l = \frac{L}{\rho V^2 b} = [2\pi(F + iG) + i\pi k]\bar{\alpha}e^{i\omega t} \qquad (7)$$

The term inside the square brackets can be considered the lift transfer function, which accounts for the difference between the unsteady and quasi-steady airloads.

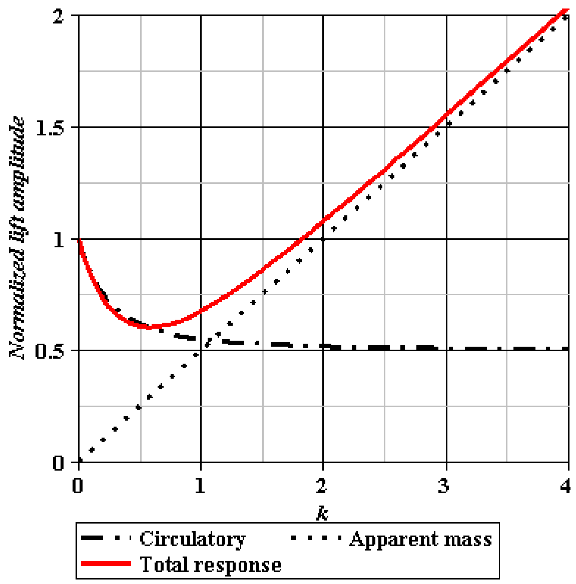

Fig. 6 Normalized Lift Amplitude

The first term inside the brackets is the circulatory term and the second term is the apparent mass contribution, which is proportional to the reduced frequency and leads the forcing by a phase angle of $\pi/2$. The noncirculatory or apparent mass terms arise from the $\partial\phi/\partial t$ term and account for the

pressure forces required to accelerate the fluid in the vicinity of the airfoil.
The normalized lift amplitude is

$$\frac{|c_l|}{2\pi|\bar{\alpha}|} = (F + iG) + i\frac{k}{2} \tag{8}$$

The lift amplitude and phase of lift for pure angle of attack oscillations are presented in fig. 6 and fig. 7, where the significance of the apparent mass contribution to both the amplitude and phase can be appreciated.

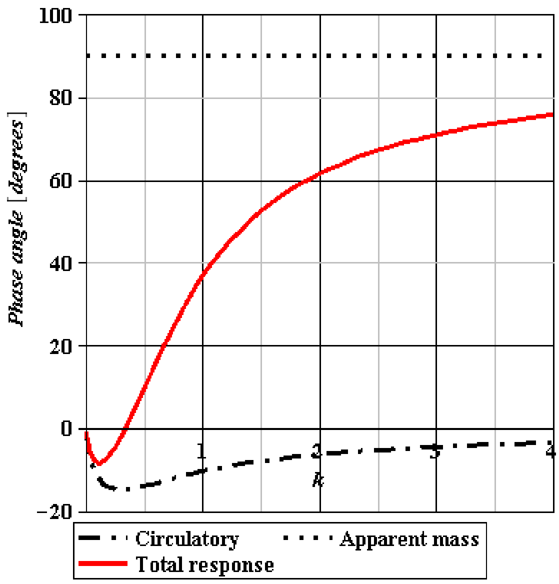

Fig. 7 Phase Angle

At lower values of reduced frequency, the noncirculatory terms dominate the solution. At higher values of reduced frequency, the apparent mass forces dominate.

Pure plunging oscillations

For a harmonic plunging motion such as be contributed by blade flapping the forcing is $h = \bar{h}e^{i\omega t}$ so that $\dot{h} = i\omega\bar{h}e^{i\omega t}$ and $\ddot{h} = -\omega^2\bar{h}e^{i\omega t}$. Substituting into the expression for the lift and solving for the lift coefficient gives

$$c_l = \left[2\pi k(iF - G) - \pi k^2\right]\frac{\bar{h}}{b}e^{i\omega t} \tag{9}$$

The complete term inside the square brackets can be considered as the lift transfer function. The circulatory part of the lift response leads the forcing displacement h by a phase angle of $\pi/2$. Also, the apparent mass force leads the circulatory part of the response by a phase angle of $\pi/2$ or the forcing by a phase angle of π. The corresponding pitching moment about mid-chord for this case is

$$c_{m1/2} = \left(\frac{\pi}{4}\right)k^2\frac{\bar{h}}{b}e^{i\omega t} \tag{10}$$

The results are plotted as the first harmonic normalized amplitude of the lift and pitching moment about the ¼-chord and their corresponding phase angles as functions of reduced frequency.

Pitching oscillations

For harmonic pitc oscillations, additional terms involving pitch rate $\dot{\alpha}$ appear in the equations for the aerodynamic response. The forcing is given by $\alpha = \bar{\alpha}e^{i\omega t}$ and the pitch rate by $\dot{\alpha} = i\omega\bar{\alpha}e^{i\omega t}$. In this case, the lift coefficient is

$$\begin{aligned} c_l &= 2\pi[F(1 + ik) + G(i - k)]\bar{\alpha}e^{i\omega t} + \\ &+ \pi k\left(i - \frac{k}{2}\right)\bar{\alpha}e^{i\omega t} \end{aligned} \tag{11}$$

The lift amplitude initially decreases with increasing k because of the effects of the shed wake and then, for $k > 0.5$ begins to increase, as the apparent mass forces begin to dominate the airloads. This is also shown by the phase angle, which exhibits an increasing lead for $k > 0.3$.

Von Karman and Sear analyzed the problem of a thin airfoil moving through a sinusoidal vertical gust field, where the gust can be considered as an upwash velocity that is uniformly convected by the free stream. The forcing function in this case is

$$w_g(x,t) = \sin\left(\omega_g t - \frac{\omega_g x}{V}\right) \tag{12}$$

where ω_g is the gust frequency. If the gust is referenced to the airfoil leading edge, then $x = 0$ and $w_g(x,t)$ becomes $w_g(x,t) = \sin(\omega_g t)$ and if the gust is referenced to the mid-chord, then $x = b = c/2$ and

$$\frac{\omega_g x}{V} = \frac{\omega_g \frac{c}{2}}{V} = \frac{\omega_g c}{2V} = k_g \tag{13}$$

Therefore,

$w_g(t) = \sin(\omega_g t)\cdot\cos(k_g) - \sin(k_g)\cdot\cos(\omega_g t)$,

which is equivalent to a phase shift. In this case the lift coefficient can be written as

$$c_l = 2\pi\left(\frac{w_0}{V}\right)S(k_g) \tag{14}$$

where $S(k_g)$ si known as Sear's function and the gust encounter frequency, k_g, is given by

$$k_g = \frac{2\pi V}{\lambda_g} \tag{15}$$

and λ_g is the wavelength of the gust.

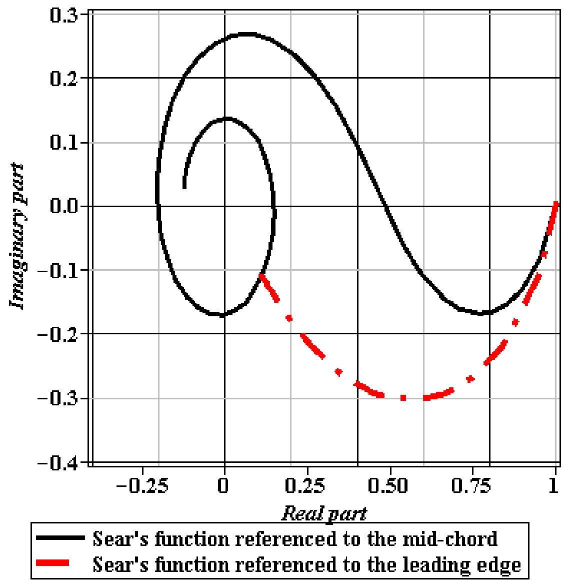

Fig. 8 Sear's Function

In terms of Bessel functions, Shear's function is given by

$S(k_g) = [J_0(k_g) - iJ_1(k_g)]C(k_g) + iJ_1(k_g)$

The terms of real and imaginary parts are

$$\begin{cases} Real\ S(k_g) = F(k_g)\cdot J_0(k_g) + G(k_g)\cdot J_1(k_g) \\ \mathrm{Im}\ S(k_g) = G(k_g)\cdot J_0(k_g) - F(k_g)\cdot J_1(k_g) + \\ \qquad + J_1(k_g) \end{cases}$$

If the gust is referenced to the leading edge of the airfoil, the function will be called S' an can be written as

$$\begin{cases} Real\ S'(k_g) = Real\ S(k_g)\cdot\cos(k_g) + \\ \qquad + \mathrm{Im}\ S(k_g)\cdot\sin(k_g) \\ \mathrm{Im}\ S'(k_g) = -Real\ S(k_g)\cdot\sin(k_g) + \\ \qquad + \mathrm{Im}\ S(k_g)\cdot\cos(k_g) \end{cases} \tag{16}$$

The two results are plotted in Fig. 8.The peculiar spiral shape of the S transfer function arises only when the gust front is referenced to the mid-chord of the airfoil.

CONCLUSIONS

Theodorsen's lift deficiency approach has found use in many problems in both fixed-wing and helicopter aeroelasticity. However, for a rotor analysis, Theodorsen's theory is somewhat less useful because the nonsteady value of velocity at the blade elements,

$V = U_T(y, \psi)$, that means that the argument k (the reduced frequency) is an ambiguous parameter. Wagner has obtained a solution for so-called indicial lift on a thin airfoil undergoing a transient step change in AoA in an incompressible flow. The variable s represents the distance traveled by the airfoil in semi-chords. The apparent mass contribution for a step imput appears as a Dirac-delta function $\delta(t)$. In Wagner's problem, the aerodynamic center is at mid-chord (at $s = 0$) and moves immediately to the ¼-chord for $s > 0$. Although the Wagner function is known exactly, its evaluation is not in a convenient analytic form, therefore it is usually replaced by a simple exponential or algebraic approximation.

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