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MATHEMATICAL MODELS USED TO STUDY THE AIRCRAFT WAKE VORTICES

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Abstract: *Developing air traffic and introducing large aircrafts in use in the group of transport aircrafts has led to the necessity to optimize separation distances between aircrafts, especially near airports. These distances are imposed by airport safety and security conditions, related to the action of the wake turbulence generated by an aircraft on another. At the edge of the aircraft wings, longitudinal vortices are created by pressure differences inside the boundary layers and rotated in opposite senses. It can constitute a danger to another aircraft that flies in this wake, especially during take-off and landing. This paper presents the mathematical models used to simulate the aircraft wake vortex behavior.*

Keywords: *aircraft, wake vortex, turbulence*

1. INTRODUCTION

Research conducted on the wake turbulence of aircrafts is determined especially by a series of economic, ecological and flight safety and security demands. These vortices are responsible for resistance to advancement induced by the lift force, which represents approximately a third of the total resistance to advancement of the aircraft. Considering the actual importance of air transport, an induced decrease with only a small percentage in resistance to advancement allows an annual significant cutback in fuel consumption (a few billions of liters of fuel).

Induced resistance is an important characteristic that is taken into consideration when projecting a new wing. Therefore, the weaker the intensity of the wake turbulence, the weaker the resistance induced by the lift force, thus decreasing fuel consumption [1]. This raises the problem of decreasing the intensity and coherence of the wake turbulence of aircrafts, in order to decrease the resistance induced by the lift force [2]. In this case it is recommended to reduce the distribution of the lift force near the end of the wing, by using longer wings and decreasing the incidence or the surface towards the end of the wing.

Wake turbulence of aircrafts also has a negative ecological consequence, because of its role in the dispersion of polluting particles in the atmosphere and through the contribution of the artificial cloud created by condensation, to the energetic balance of the planet. The persistence of the artificial cloud created in the air is directly connected to the duration of vortices inside wakes.

The most important consequence of wake turbulence consists of the risk of air incidents due to the encounter between an aircraft and the wake generated by a preceding aircraft [3].

This situation is more frequent in the take-off and landing phases, when aircrafts are encountered, during movement, one behind the other, at a small enough distance and when interaction phenomena between the wake and the ground can lead to an increase of the intensity of wake vortices and, thus, to a probability of air accident production with devastating consequences.

2. RISK ASSESSMENTS OF ACCIDENTS PRODUCED AS A RESULT OF THE AIRCRAFT WAKE VORTEX

The main effect of wake turbulence on an aircraft is the induced rolling motion that can even lead to an overturn. These phenomena can become extremely dangerous during the take-off or landing phases when the aircraft moves at a low altitude and can't recover. Aircrafts that have small wing spans are the most affected [4]. The effect of the wake depends on various factors, of which we remind the weight and the span of the aircraft that enters in the action range of the wake and the relative position between the aircraft and the wake turbulence. Due to the action of the wake turbulence, numerous air accidents happened along time:

- 30 May 1972 - The McDonnell Douglas DC-9-14 Aircraft crashed on the Southwest International Airport in Fort Worth, Texas, being affected by the wake turbulence generated by a McDonnell Douglas DC-10 aircraft. Following this incident, a series of regulations regarding the minimal separation distances between aircrafts have been introduced;
- 20 September 1999 - a JAS39 Gripen aircraft passed through the wake turbulence of another aircraft of larger proportions during a military maneuver, crashing in a lake;
- 12 November 2001 - an Airbus A300 aircraft crashed on New York airport, under the action of the wake turbulence generated by a Boeing 747 aircraft which had taken off two minutes before;
- 4 November 2008 - LearJet 45 XC-VMC aircraft crashed near the international airport in Mexic. Subsequently, it was proven that this incident was due to the action of the wake turbulence of a Boeing 767 aircraft;
- 25 February 2009 - the Boeing 737-800 aircraft of the Turkish Airlines company crashed near the Schiphol airport in Amsterdam, being affected by the wake turbulence of a Boeing 757 aircraft, that had landed on the same landing run two minutes before;
- 2 March 2009 - according to press information, the Control Tower of the Henri Coanda airport in Otopeni discovered in due time that the landing trajectory of the ATR aircraft of the TAROM company intersected that of a Boeing 727 aircraft of the same company. At the last moment, the employees of the control tower have cancelled the landing procedure, and the plane was forced to postpone landing, in order to avoid passing through the action range of the wake turbulence of the first aircraft.

In order to avoid air incidents caused by the action of wake turbulence a series of regulations have been established to define the minimal time intervals between two landings/take-offs of two aircrafts, depending on their proportions, so that the wake of the first aircraft doesn't affect the second aircraft significantly (fig.1). For example, no other aircraft should take-off or land behind a Boeing 747, no sooner than two minutes, which corresponds to a distance of approximately 7,2 km between the aircrafts. This distance becomes greater if the following aircraft is from a smaller category [5].

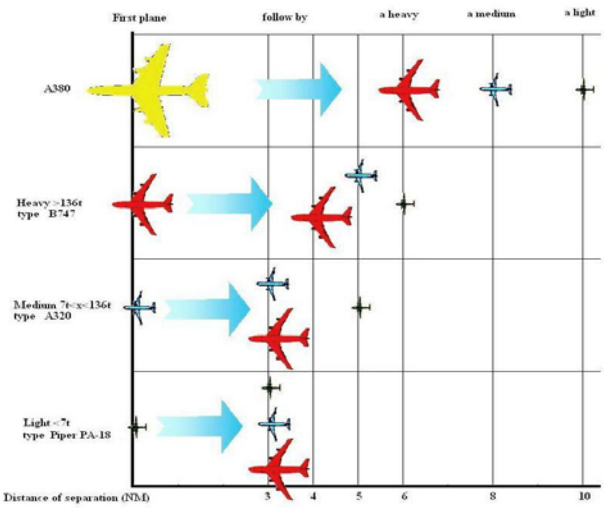


Fig. 1. Distance of separation between two aircrafts

The minimal time intervals established by the air authorities led to a limitation in air traffic. Therefore, modern airports are dealing with a highly important problem that is optimizing air traffic so that it allows the circulation of a larger number of aircrafts on the take-off/landing run, but without endangering them [5].

Research on wake dynamics have been the object of important programs financed both by the European Union and by other international organizations.

3. MODELS FOR WAKE VORTEX

3.1 Rankine vortex. A first approach to longitudinal vortex is the two-dimensional isolated vortices type Rankine. This vortex is a classic model characterized by a rotation in the vortex core. It corresponds to a constant vortices (ω) distribution in a cylindrical tube infinite and with a radius r_0. The vortices are equal to zero outside of the tube (fig. 2).

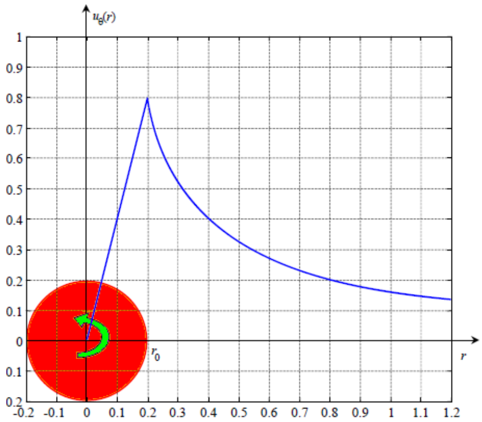


Fig. 2. Distribution of the tangential velocity for a Rankine vortex

The vorticity for the Rankine vortex will have the distribution:

$$\omega(r)=\begin{cases}\omega_0=\dfrac{\Gamma_0}{\pi r_0^2}, 0<r\le r_0\\ 0, r_0<r\end{cases} \tag{1}$$

The tangential speed for the Rankine vortex will have the relation:

$$u_\theta(r)=\begin{cases}\dfrac{1}{r}\displaystyle\int_0^r \omega_0 r' dr', 0<r\le r_0,\\ \dfrac{1}{r}\displaystyle\int_0^{r_0} \omega_0 r' dr', r_0<r.\end{cases} \tag{2}$$

$$u_\theta(r)=\begin{cases}\dfrac{\omega_0 r}{2}=\dfrac{\Gamma_0 r}{2\pi r_0^2}, 0<r\le r_0,\\ \dfrac{\omega_0 r_0^2}{2r}=\dfrac{\Gamma_0}{2\pi r}, r_0<r.\end{cases} \tag{3}$$

The Rankine vortex model which is presented here consists of a solid rotation of the heart of the vortex and a potential flow. In this model the viscosity isn't taken into account and also this model is not an exact solution of Navier-Stokes equations [6].

3.2 Lamb-Oseen vortex. In analyzing the influence of the viscosity on the vortex dynamics we used another longitudinal vortex, Lamb-Oseen vortex model. This situation allows us to obtain an analytic solution. The flow of a single vortex isolated as Lamb-Oseen admits a revolution symmetry. We chose also cylindrical coordinates. We used Navier - Stokes equations to solve this problem [7].

The vorticity for a vortex will have this expression:

$$\omega_x(r,T)=\frac{\Gamma}{\pi r_0^2(1+T)}e^{-\frac{r^2}{r_0^2}\frac{1}{1+T}}. \tag{4}$$

where we use the reduced time T, and we note the viscous time with t_v:

$$T=\frac{t}{t_v}, t_v=\frac{r_0^2}{4v}. \tag{5}$$

Also, the tangential velocity (fig. 3) of a vortex will have this expression:

$$u_\theta(r,T)=\frac{\Gamma}{2\pi r}\left[1-e^{-\frac{r^2}{r_0^2}\frac{1}{1+T}}\right]. \qquad (6)$$

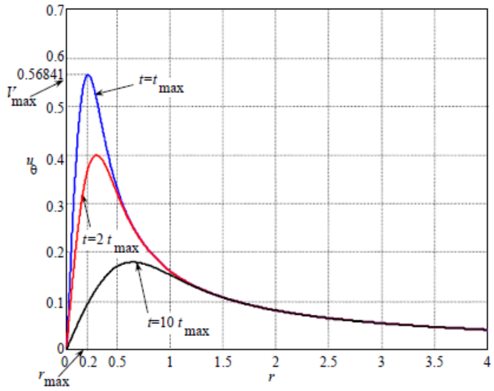


Fig. 3. The tangential velocity for a Lamb-Oseen vortex

The Lamb-Oseen vortex is a simple model of longitudinal vortex frequently used to model the wake of the aircraft. It provides a good numerical representation of longitudinal vortices.

3.3 Lamb-Chaplygin model for wake-vortex pair. Preceding models are used for solitary vortices. The non-linearity of the Navier-Stokes equations imposes some problems in the case of the superposition of elementary isolated vortices. The model of Lamb-Chaplygin takes into account the both the high wake vortices. Besides, in this model, it is not held counts of the viscosity, what removes considerably results of the reality.

Lamb-Chaplygin model consists of a counter-rotating wake-vortex pair with the vorticity concentrated in the circle with the radius R (fig. 4):

$$\omega(r,\theta)=\begin{cases}-\dfrac{\mu_1^2}{R^2}\psi(r,\theta), & r\le R\\ 0, & r>R\end{cases}, \qquad (7)$$

where:

U – velocity of vortex-pair propagation;

J_0, J_1 – Bessel functions;

$\mu_1=3.8317$, the first solution of J_1;

$$\psi(r,\theta)=\begin{cases}-\dfrac{2U\cdot R}{\mu_1 J_0(\mu_1)}J_1\left(\mu_1\dfrac{r}{R}\right)\sin\theta, & r\le R\\ -U\cdot r\left(1-\dfrac{R^2}{r^2}\right)\sin\theta, & r>R\end{cases}. \qquad (8)$$

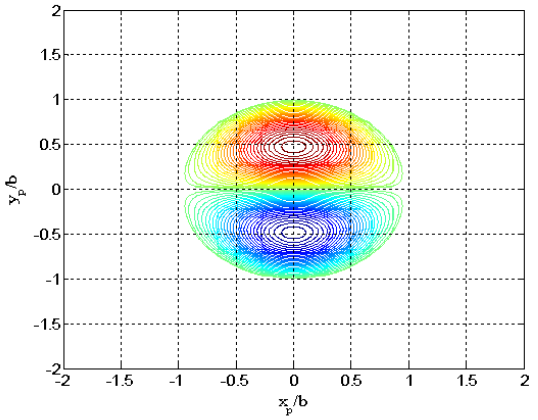


Fig. 4. Distribution of the vorticity for a Lamb-Chaplygin vortex pair

3.4 Counter-rotating vortices using Lamb-Oseen model. To obtain a configuration of a wake of aircraft we can choose the Lamb-Chaplygin model or use the superposition of two simple vortices (Rankine or Lamb-Oseen) model (fig. 5).

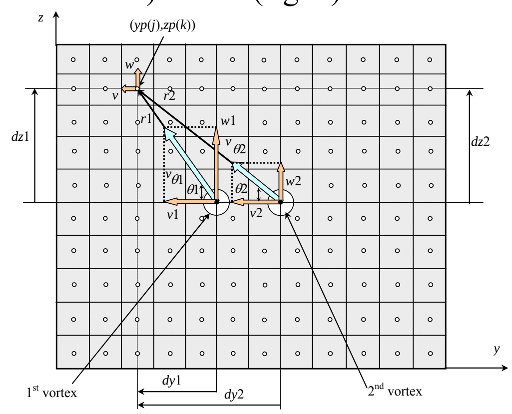
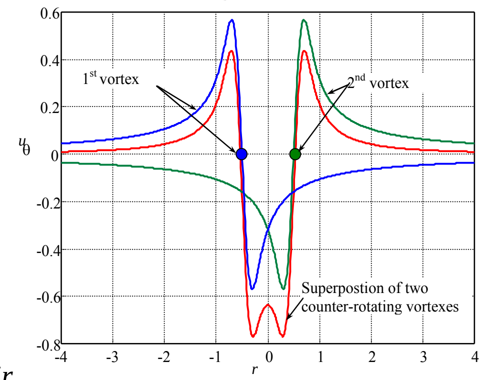


Fig. 5. Velocity field in the case of counter-rotating vortex



pair

Fig. 6. Velocity distribution in the case of counter-rotating vortex pair

The solution obtained by using of superposition of two simple vortices is not a solution of the equations of Navier - Stokes due to the non-linearity of these equations.

After superposition of two vortices we should use a calculation of adaptation to remove this defect [10]. Fig. 6 and 7 present the velocity and pressure distribution obtained using the superposition methods of two counter-rotating Lamb-Oseen vortices.

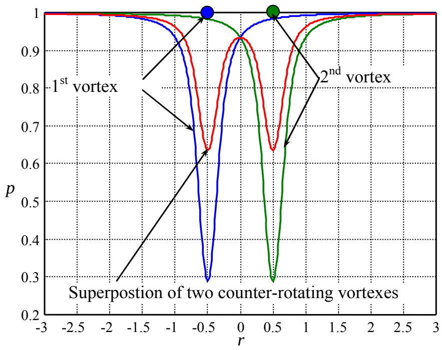


Fig. 7. Pressure distribution in the case of counter-rotating vortex pair

Solutions are defined within calculation, also to have solutions that correspond to the reality, this implements a mathematical model that will interact with the solution proposed for the closer to reality. After a sufficient number of iterations of the simulation of adaptation, speeds and pressures field obtained can be considered as the Navier Stokes equations solution [11].

3.5 Counter-rotating vortices using rows of vortices model. The method of modeling a pair of counter-rotating vortices by the superposition method describes the flow in the infinite medium. The numerical simulations are often done in a finite computation field. In this case the rows of vortices are the most appropriated method [9].

To create the rows of vortices are used Lamb-Oseen vortices (fig. 8). For a single vortex with the coordinates of its heart in the plane Oyz, (y_c, z_c), the velocity field has the following distribution:

$$\begin{cases} v(y,z) = -\dfrac{\Gamma_0}{2\pi}\dfrac{z-z_c}{r^2}\left[1-e^{-(r/r_c)^2}\right] \\ w(y,z) = \dfrac{\Gamma_0}{2\pi}\dfrac{y-y_c}{r^2}\left[1-e^{-(r/r_c)^2}\right] \end{cases}, \quad (9)$$

where $r = \sqrt{(y-y_c)^2 + (z-z_c)^2}$.

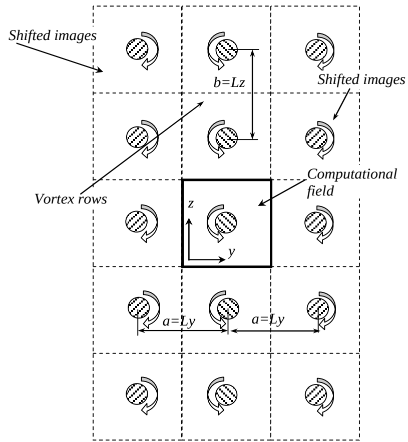


Fig. 8. Rows of vortices

We consider a vertical row of vortices at a distance. To determine the velocity distribution for this row using the stream function as follows:

$$f(\xi) = -\frac{\Gamma_0}{2\pi}\ln(\xi - \xi_c), \quad (10)$$

where $\xi = y + zi$ et $\xi_c = y_c + z_c i$.

After computing for $n \to \infty$ vortices we get the rows of vortices:

$$\begin{cases} v(y,z) = -\dfrac{\Gamma_0}{2b}\dfrac{\sin[2\pi(z-z_c)/b]}{\cosh[2\pi(y-y_c)/b] - \cos[2\pi(z-z_c)/b]} \\ w(y,z) = \dfrac{\Gamma_0}{2b}\dfrac{\sinh[2\pi(y-y_c)/b]}{\cosh[2\pi(y-y_c)/b] - \cos[2\pi(z-z_c)/b]} \end{cases}, \quad (11)$$

In the middle row we must remove the solution corresponding computational domain:

$$\begin{cases} v(y,z) = -\dfrac{\Gamma_0}{2\pi}\dfrac{z-z_c}{r^2} \\ w(y,z) = \dfrac{\Gamma_0}{2\pi}\dfrac{y-y_c}{r^2} \end{cases}. \quad (12)$$

It has been added m shifted images in the horizontal direction. The final solution is:

$$S(y_c,z_c)=S_o(y_c,z_c)+S_r(y_c,z_c)-$$
$$-S_p(y_c,z_c)-\sum_m S_r(y_c\pm a\cdot m,z_c) \quad (13)$$

where:

S – final solution;

S_0– solution for the central vortex pair (9);

S_r – solution corresponding to the row of vortex (11);

S_p – solution corresponding to computational field (12).

5. CONCLUSIONS & ACKNOWLEDGMENT

The presented models can be used as initial condition for the numerical simulations of the behavior of the isolated wake vortex or of the wake vortex counter-rotating pair [8]. The numerical simulation can be realized using the Direct Numerical Simulation or Large Eddy Simulation methods [12].

These methods are based on finding a solution through the method of finite volume of fundamental equations of turbulent fluid flow, the Navier-Stokes equations. The Direct Numerical Simulation (DNS) method of modeling turbulent flow has the advantage of being very precise, but it's necessary to realize simulations on a very large number of calculus points, which obviously involves using a high calculus power.

The Large Eddy Simulation (LES) method of modeling turbulent flow consists of directly simulating large turbulent structures, the small ones being modeled by specific methods. The reasoning of this method is based on the fact that large turbulent structures are directly influenced by the geometrical characteristics of the studied situation, while small structures have an universal character, and the errors introduced by modelling them are insignificant. The advantage of this method is that it offers results as precise as those obtained through the direct numerical simulation method, but using a smaller amount of calculus numbers, therefore a smaller calculus power.

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