"HENRI COANDA"
AIR FORCE ACADEMY
ROMANIA

"GENERAL M.R. STEFANIK"
ARMED FORCES ACADEMY
SLOVAK REPUBLIC

INTERNATIONAL CONFERENCE of SCIENTIFIC PAPER
AFASES 2014
Brasov, 22-24 May 2014

UNSTEADY AERODYNAMIC MODEL FOR AN AIRFOIL WITH TIME DEPENDENT BOUNDARY CONDITIONS

Constantin ROTARU*, Raluca Ioana EDU*, Mihai ANDRES-MIHAILA*

*Faculty of Mechatronics and Armament Integrated Systems, Military Technical Academy, Bucharest, Romania

Abstract: *This paper presents a mathematical model of an unsteady fluid flow around an airfoil where the time dependency is introduced through the boundary conditions. The methods of solution that were developed for these models included the treatment of the zero normal flow on a solid surface and the use of the unsteady Bernoulli equation. As a result of the nonuniform motion, the wake becomes more complex than in the corresponding steady flow case and therefore the path along which the airfoil moves was assumed to be prescribed. One of the more difficult aspects of the unsteady problem is the modeling of the vortex wake's shape and strength, which depend on the time history of the motion. In the paper the wake shed from the trailing edge of the lifting surfaces was modeled by vortex distributions.*

Keywords: *unsteady aerodynamics, vortex flow, airfoil circulation*

1. INTRODUCTION

In an incompressible and irrotational fluid flow, the velocity field can be obtained by solving the continuity equation. However, the incompressible continuity equation does not directly include time-dependent terms and the time dependency is introduced through the boundary conditions. The methods of solution for steady flows can be used with only small modifications that include the treatment of the zero normal flow on a solid surface boundary conditions and the use of the unsteady Bernoulli equation. As a result of the nonuniform motion, the wake becomes more complex than in the corresponding steady flow case and it should be properly accounted for.

The unsteady motion of the surface on which the "zero normal flow" boundary condition is applied, is described in a body-fixed coordinate system (x, y, z) and the motion of the origin of this coordinate system (Fig. 1) is then prescribed in an inertial frame of reference (X, Y, Z).

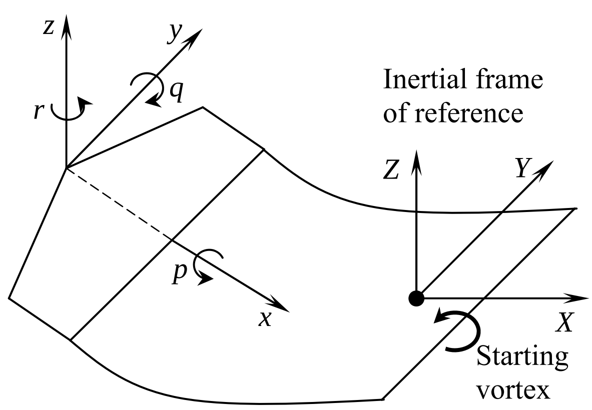

Fig. 1. Coordinate systems

At the $t > 0$, the relative motion of the origin of the body fixed frame of reference is prescribed by its location $R_0(t) = (X_0, Y_0, Z_0)$ and the instantaneous orientation $\Theta(t) = (\varphi, \theta, \psi)$, where (φ, θ, ψ) are the Euler rotation angles [1].

The fluid surrounding the body is assumed to be inviscid, irrotational and incompressible over entire flow field, excluding the body's solid boundaries and its wake. Therefore, a velocity potential $\Phi(X, Y, Z)$ can be defined in the inertial frame and the continuity equation in this frame of reference becomes $\nabla^2\Phi = 0$ and the boundary condition requiring zero normal velocity across the body's solid boundaries is

$$(\nabla\Phi + \vec{v}) \cdot n = 0 \tag{1}$$

where $\vec{v}$ is the surface velocity and $\vec{n}(x, y, z, t)$ is the unity vector normal to this moving surface (v is defined with minus sign so that the undisturbed flow velocity will be positive in the body's frame of reference).

The location and orientation of $\vec{n}$ can vary with time, so, the time dependency of equation $\nabla^2\Phi = 0$ is introduced through the boundary condition. The second boundary condition requires that the flow disturbance due to the body's motion through the fluid, should diminish far from the body,

$$\lim_{|R - R_0| \to \infty} \nabla\Phi = 0 \tag{2}$$

where $R = (X, Y, Z)$.

On the other hand, the Kelvin equation could be an additional condition for the unsteady flow, that can be used to determine the stream wise strength of the vorticity shed into a wake, so, the circulation Γ around a fluid curve enclosing the body and its wake is conserved,

$$d\Gamma / dt = 0 \tag{3}$$

Because of the boundary condition this problem becomes time dependent and it could be solved easier in the body-fixed coordinate system [2]. A transformation from (X, Y, Z) coordinate system to (x, y, z) coordinate system should include the translation and the rotation of the (x, y, z) system and may have the following form

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\varphi t) & \sin(\varphi t) \\ 0 & -\sin(\varphi t) & \cos(\varphi t) \end{pmatrix} \times \begin{pmatrix} \cos(\varphi t) & 0 & -\sin(\varphi t) \\ 0 & 1 & 0 \\ \sin(\varphi t) & 0 & \cos(\varphi t) \end{pmatrix} \times \begin{pmatrix} \cos(\varphi t) & \sin(\varphi t) & 0 \\ -\sin(\varphi t) & \cos(\varphi t) & 0 \\ 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} X - X_0 \\ Y - Y_0 \\ Z - Z_0 \end{pmatrix}$$

The kinematic velocity $\vec{v}$ of the undisturbed fluid due to the motion of the airfoil as viewed in the body frame of reference is given by

$$\vec{v} = -(\vec{V}_0 + \vec{v}_{rel} + \vec{\Omega} \times \vec{r}) \tag{4}$$

where

$$\begin{cases} \vec{V}_0 = (\dot{X}_0, \dot{Y}_0, \dot{Z}_0) \\ \vec{v}_{rel} = (\dot{x}, \dot{y}, \dot{z}) \\ \vec{\Omega} = (p, q, r) \\ \vec{r} = (x, y, z) \end{cases} \tag{5}$$

At any moment the continuity equation is independent of the coordinate system orientation and the mass is conserved [3]. Therefore, the quantity $\nabla^2\Phi$ is independent of the instantaneous coordinate system and the continuity equation in terms of (x, y, z) remains unchanged, $\nabla^2\Phi = 0$.

2. WAKE SHAPE

The zero velocity normal to the solid surface boundary condition in the body frame is

$$(\nabla\Phi - \vec{V}_0 - \vec{v}_{rel} - \vec{\Omega} \times \vec{r}) \cdot \vec{n} = 0 \tag{6}$$

in (x, y, z) coordinates.

In the case of more complex flow field, when the modeling of nonzero velocity components across the boundaries is desired, a transpiration velocity, V_n can be added, so, the above equation becomes

$$\left(\nabla\Phi - \vec{V}_0 - \vec{v}_{rel} - \vec{\Omega}\times\vec{r}\right)\cdot\vec{n} = V_n \tag{7}$$

For incompressible flows the instantaneous solution is independent of time derivatives, therefore the steady-state solution techniques can be used to approach the time dependent problem by substituting at each moment, the instantaneous boundary condition

$$\left(\nabla\Phi + \vec{v}\right)\cdot n = 0 \tag{8}$$

For lifting flow conditions, the magnitude of circulation depends on the wake shape and on the location of the wake shedding line [4]. Taking into consideration that the wake is force free, the Kutta-Jukovski theorem states that

$$V_\infty \times \gamma_W = 0 \tag{9}$$

so, when the wake is modeled by a vortex distribution of strength γ_W the velocity V_∞ should be parallel to the circulation vector γ_w.

Solution of the equation $\nabla^2\Phi = 0$ in (x, y, z) coordinates, provides the velocity potential and the velocity components [5]

$$\begin{cases} u = \partial\Phi/\partial X \\ v = \partial\Phi/\partial Y \\ w = \partial\Phi/\partial Z \end{cases} \tag{10}$$

The resulting pressure can be computed by the Bernoulli equation

$$\frac{p_\infty - p}{\rho} = \frac{1}{2}(\nabla\Phi)^2 + \frac{\partial\Phi}{\partial t} = \\ = \frac{1}{2}\left[\left(\frac{\partial\Phi}{\partial X}\right)^2 + \left(\frac{\partial\Phi}{\partial Y}\right)^2 + \left(\frac{\partial\Phi}{\partial Z}\right)^2\right] + \frac{\partial\Phi}{\partial t} \tag{11}$$

The time derivative in the (x, y, z) system is

$$\left(\frac{\partial}{\partial t}\right)_{inertial} = \\ -\left(\vec{V}_0 + \vec{\Omega}\times\vec{r}\right)\cdot\left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z},\right) + \left(\frac{\partial}{\partial t}\right)_{body} \tag{12}$$

therefore, the pressure difference $(p_\infty - p)/\rho$ has the form

$$\frac{p_\infty - p}{\rho} = \frac{1}{2}\left[\left(\frac{\partial\Phi}{\partial x}\right)^2 + \left(\frac{\partial\Phi}{\partial y}\right)^2 + \left(\frac{\partial\Phi}{\partial z}\right)^2\right] - \\ -\left(\vec{V}_0 + \vec{\Omega}\times\vec{r}\right)\cdot\nabla\Phi + \frac{\partial\Phi}{\partial t}$$

The magnitude of the velocity $\nabla\Phi$ is independent of the frame of reference.

3. PLAT PLATE MODEL

For a flat plate at an angle of attack α moving at a constant velocity U_∞ in the negative X direction (Fig. 2) the translation of the origin $\vec{V}_0$, the rotation $\vec{\Omega}$ and the normal vector $\vec{n}$, are

$$\begin{aligned} &\vec{V}_0 = \left(\dot{X}_0, \dot{Y}_0, \dot{Z}_0\right) = \left(-U_\infty, 0, 0\right) \\ &\vec{\Omega} = (0, 0, 0) \\ &\vec{n} = (\sin\alpha, 0, \cos\alpha) \end{aligned} \tag{13}$$

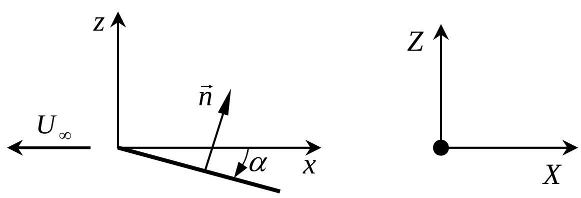

Fig. 2. Translation of a flat plate

The boundary condition requiring zero normal velocity across the plate is

$$\left(\nabla\Phi - \vec{V}_0 - \vec{\Omega}\times\vec{r}\right)\cdot\vec{n} = 0 \tag{14}$$

or

$$\left(\frac{\partial\Phi}{\partial x}+U_\infty, 0, \frac{\partial\Phi}{\partial z}\right)\cdot(\sin\alpha, 0, \cos\alpha)=0$$

The above scalar product has the form

$$\left(\frac{\partial\Phi}{\partial x}+U_\infty\right)\sin\alpha+\frac{\partial\Phi}{\partial z}\cos\alpha=0 \tag{15}$$

therefore

$$\frac{\partial\Phi}{\partial z}=-\left(\frac{\partial\Phi}{\partial x}+U_\infty\right)\tan\alpha \tag{16}$$

If the airfoil (plate) is represented by a lumped-vortex element with the vortex placed at the quarter chord (Fig. 3), the Kutta condition is satisfied.

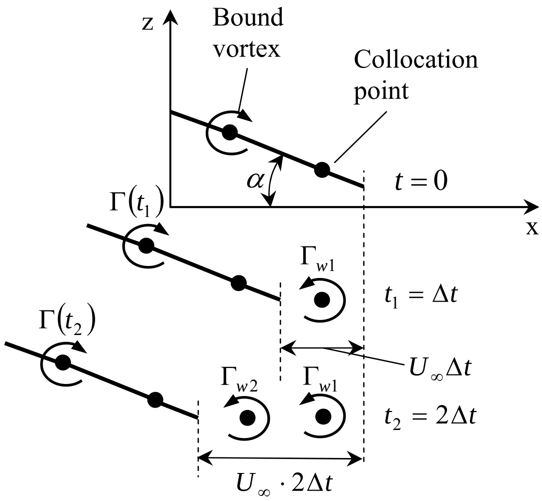

Fig. 3. Development of the wake vortex

The concentrated wake vortex has to be placed along the path traveled by the trailing edge [6].

If the wake vortex is placed at the middle of this path, then the zero normal flow boundary condition at the plate's three-quarter point is

$$-\frac{\Gamma(t_1)}{2\pi\frac{c}{2}}+\frac{T_{w1}}{2\pi\left(\frac{c}{4}+\frac{U_\infty\Delta t}{2}\right)}=-U_\infty\sin\alpha \tag{17}$$

This equation can be rewritten in the form

$$w_{body}+w_{wake}+U_\infty\sin\alpha=0 \tag{18}$$

which indicates that the sum of the normal velocity induced by the airfoil, w_{body}, by the wake, w_{wake}, and by the free stream must be zero.

On the other hand, an additional equation could be obtained from the Kelvin condition $(d\Gamma/dt=0)$, namely

$$\Gamma(t_1)+\Gamma_{w1}=0 \tag{19}$$

The above set of equations with the unknowns $\Gamma(t_1)$ and Γ_{w1} gives the following solution

$$\begin{cases}\Gamma(t_1)=\dfrac{U_\infty\sin\alpha}{\dfrac{1}{2\pi}\left(\dfrac{1}{\frac{c}{2}}+\dfrac{1}{\frac{c}{4}+\frac{1}{2}U_\infty\Delta t}\right)}\\[2ex]\Gamma_{w1}=-\dfrac{U_\infty\sin\alpha}{\dfrac{1}{2\pi}\left(\dfrac{1}{\frac{c}{2}}+\dfrac{1}{\frac{c}{4}+\frac{1}{2}U_\infty\Delta t}\right)}\end{cases} \tag{20}$$

After the second time step, $t_2=2\Delta t$, the airfoil is in a new location. For high Reynolds number flows, vortex decay is negligible and therefore the strength Γ_{w1} will not change with time. At $t_2=2\Delta t$ the two equations describing the zero normal flow boundary condition and the Kelvin condition are:

$$\begin{cases}-\dfrac{\Gamma(t_2)}{2\pi\frac{c}{2}}+\dfrac{\Gamma_{w2}}{2\pi\left(\frac{c}{4}+\frac{1}{2}U_\infty\Delta t\right)}+\\[2ex]+\dfrac{\Gamma_{w2}}{2\pi\left(\frac{c}{4}+\frac{1}{2}U_\infty\Delta t+U_\infty\Delta t\right)}=-U_\infty\sin\alpha\\[2ex]\Gamma(t_2)+\Gamma_{w2}+\Gamma_{w2}=0\end{cases}$$

This set is solved for $\Gamma(t_2)$ and Γ_{w2}, while Γ_{w1} is known from the previous calculation at $t=t_1$. At $t=3\Delta t$ the two equations can be written in a similar manner

$$\begin{cases}-\dfrac{\Gamma(t_3)}{2\pi\frac{c}{2}}+\dfrac{\Gamma_{w3}}{2\pi\left(\frac{c}{4}+\frac{1}{2}U_\infty\Delta t\right)}+\\[2ex]\dfrac{\Gamma_{w2}}{2\pi\left(\frac{c}{4}+\frac{3}{2}U_\infty\Delta t\right)}+\dfrac{\Gamma_{w1}}{2\pi\left(\frac{c}{4}+\frac{5}{2}U_\infty\Delta t\right)}=\\[2ex]=-U_\infty\sin\alpha\\[1ex]\Gamma(t_3)+\Gamma_{w3}+\Gamma_{w2}+\Gamma_{w1}=0\end{cases}$$

This set is solved for $\Gamma(t_3)$ and Γ_{w3}, while Γ_{w2} and Γ_{w1} are known from the previous calculation at $t = t_2$.

The values of $\Gamma(t_i)$ and Γ_{wi} are found from the following set of equations written in the matrix form

$$\begin{bmatrix} -\dfrac{1}{2\pi\dfrac{c}{2}} & \dfrac{1}{2\pi\left(\dfrac{c}{4}+\dfrac{1}{2}U_\infty\Delta t\right)} \\ 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} \Gamma(t_i) \\ \Gamma_{wi} \end{bmatrix} = \begin{bmatrix} -U_\infty \sin\alpha - \displaystyle\sum_{j=1}^{i-1} \frac{\Gamma_{w(i-j)}}{2\pi\left[\dfrac{c}{4}+(2j+1)\dfrac{U_\infty\Delta t}{2}\right]} \\ -\displaystyle\sum_{j=1}^{i-1}\Gamma_{wj} \end{bmatrix}$$

4. WORTEX DISTRIBUTION

The wake shed from the trailing edge of the lifting surfaces can be modeled by doublet or vortex distributions (Fig. 4).

If the airfoil circulation is varying continuously, then a continuous vortex sheet is shed at the trailing edge and can be approximated by a discrete vortex model where the strength of each vortex Γ_{wi} is equal to the vorticity shed during the corresponding time step Δt, such that

$$\Gamma_{wi} = \int_{t-\Delta t}^{t} \gamma_{wi}(t) U_\infty dt \qquad (21)$$

The distance and relative angle to the trailing edge are important numerical parameters and the wake vortex location should be closer to the position of the trailing edge (Fig. 5).

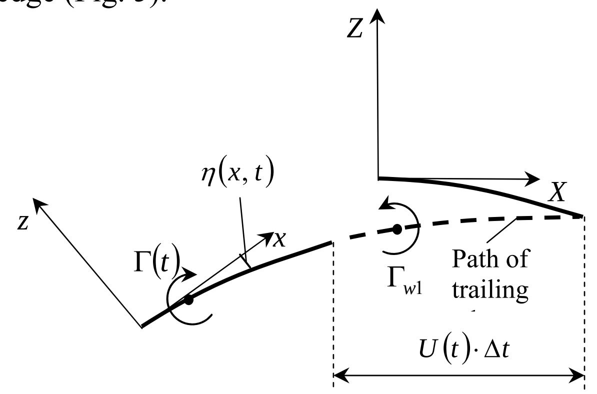

Fig. 5. Position of the discrete vortex

The placement of the discrete vortex at the middle of the interval $U(t)\Delta t$ is an approximation that underestimates the induced velocity when compared with the continuous wake vortex sheet result. A numerical approach to correct for this wake discretization error is to place the latest vortex closer to the trailing edge.

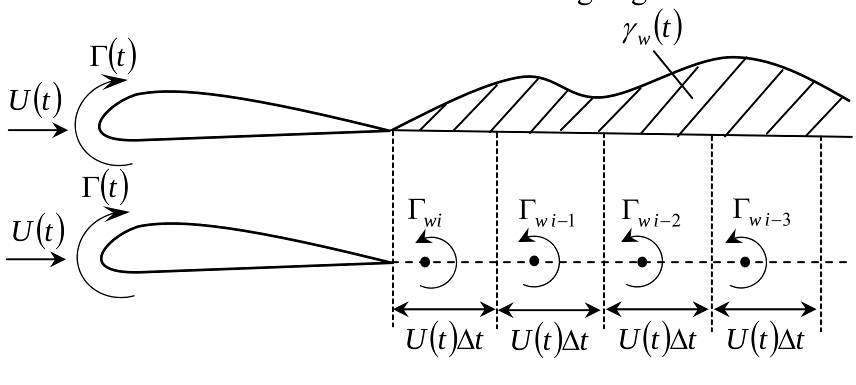

Fig. 4. Discretization of the wake's vortex distribution

The Helmholtz theorem implies that there is no vortex decay, that is if a wake vortex element is shed from the trailing edge, its strength is conserved. At each time step the combined airfoil and wake induced velocity $(u, w)_i$ is calculated and the vortex elements are moved by $(\Delta x, \Delta y) = (u, w)_i \Delta t$. The system coordinate (x, z) is selected such that the origin is placed on the path and the x coordinate axis is tangent to the path. The airfoil camberline is given in this coordinate system by $\eta(x, t)$, which is considered to be small $(\eta / c << 1)$ and the path radius of curvature is also much larger than the chord c.

5. MATHEMATICAL FORMULATION

The time-dependent version of the boundary condition requiring no normal flow across the surface is

$$\left(\nabla\Phi - \vec{V}_0 - \vec{v}_{rel} - \vec{\Omega}\times\vec{r}\right)\cdot\vec{n} = 0 \qquad (22)$$

where $\Phi = \Phi_B + \Phi_w$ is the equivalent of the steady-state velocity potential, divided into airfoil potential Φ_B and to a wake potential Φ_w, $\vec{V}_0$ is the instantaneous velocity of the coordinate system origin, $V_0 = [-U(t), 0, 0]$, $\vec{v}_{rel}$ is the relative velocity of the chordline within coordinate system (x, y, z), $v_{rel} = \left[0, 0, \frac{\partial\eta}{\partial t}\right]$, $\vec{\Omega}$ is the instantaneous rotation, $\Omega = [0, \dot{\theta}(t), 0]$ and $\vec{n}$ is the normal vector to the surface

$$n = \frac{\left[-\frac{\partial\eta}{\partial x}, 0, 1\right]}{\sqrt{\left(\frac{\partial\eta}{\partial x}\right)^2 + 1}} \qquad (23)$$

If the wake potential is known from the previous time steps then

$$\frac{\partial\Phi_B}{\partial z} = \left(\frac{\partial\Phi_B}{\partial x} + \frac{\partial\Phi_w}{\partial x} + U - \dot{\theta} z\right)\frac{\partial\eta}{\partial x} - \frac{\partial\Phi_w}{\partial z} - \dot{\theta} x + \frac{\partial\eta}{\partial t} \qquad (24)$$

On the other hand, the downwash induced by the airfoil bound circulation $\gamma(x, t)$ with assumptions presented above is

$$\frac{\partial\Phi_B}{\partial z} = -\frac{1}{2\pi}\int_0^c \frac{\varphi(x_0, t)}{x - x_0}dx_0 \qquad (25)$$

so, the time dependent equivalent of the steady-state boundary condition becomes

$$-\frac{1}{2\pi}\int_0^c \frac{\varphi(x_0, t)}{x - x_0}dx_0 = U(t)\frac{\partial\eta(x, t)}{\partial x} - \frac{\partial\Phi_w}{\partial z} - \dot{\theta}(t)\cdot x\frac{\partial\eta(x, t)}{\partial t} \qquad (26)$$

with the Kutta condition $\gamma(c, t) = 0$.

Based on the classical approach of Glauert, a similar solution to the vortex distribution is

$$\gamma(\theta, t) = 2U(t)\left[A_0(t)\frac{1 + \cos\theta}{\sin\theta} + \sum_{n=1}^{\infty} A_n(t)\sin n\theta\right]$$

where

$$A_0(t) = -\frac{1}{\pi}\int_0^\pi \frac{w(x, t)}{U(t)}d\theta$$

$$A_n(t) = \frac{2}{\pi}\int_0^\pi \frac{w(x, t)}{U(t)}\cos n\theta d\theta$$

The lift force per unit span L' and the pitching moment about the airfoil's leading edge M_0 are

$$L'(t) = \pi\rho c\left\{\left[U^2 A_0 + \frac{3c}{4}\frac{\partial}{\partial t}(U A_0)\right] + \left[+U^2\frac{A_1}{2} + \frac{c}{4}\frac{\partial}{\partial t}(UA_1) + \frac{c}{8}\frac{\partial}{\partial t}(UA_2)\right]\right\}$$

$$M_0(t) = -\rho c^2\frac{\pi}{2}\left[\frac{U^2}{2}A_0 + \frac{7c}{8}\frac{\partial}{\partial t}(U A_0) + \frac{U^2}{2}A_1 + \frac{3c}{8}\frac{\partial}{\partial t}(U A_1)\right] - \rho c^2\frac{\pi}{2}\left[-\frac{U^2}{4}A_2 + \frac{c}{8}\frac{\partial}{\partial t}(U A_2) - \frac{c}{32}\frac{\partial}{\partial t}(U A_3)\right]$$

6. NUMERICAL RESULTS

The results of the computations for a number of steps of 200, a velocity $U = 50\ m/s$ an angle of attack $\alpha = 5\cdot\pi/180$ rad and a time step $\Delta t = 1/(4\cdot U)$ are presented in fig. 6. The circulation at $t = 0$ is zero since the airfoil is still at rest. At $t > 0$ the circulation increases but is far less than the steady-state value due to the downwash of the starting vortex.

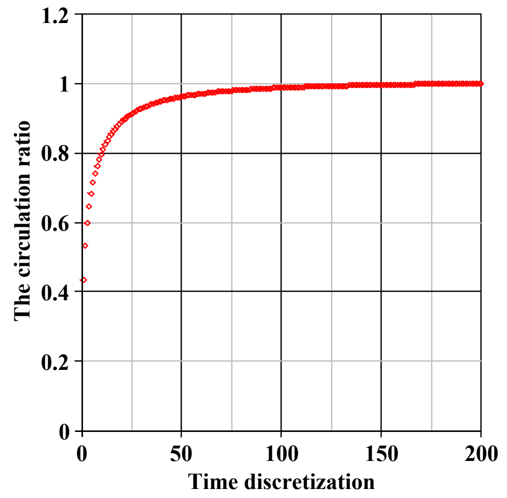

Fig. 6. The circulation ratio $\Gamma(t)/\Gamma(200)$

In the above figure was represented the ratio between $\Gamma(t)/\Gamma(200)$, where the number of time steps was 200. After approximately 200 steps this ratio reaches the value equals with the unity. To compute the lift , the small disturbance approximation $(U_\infty >> \nabla\Phi)$ is applied to the unsteady Bernoulli equation

$$\frac{p_\infty - p}{\rho} = (U_\infty, 0, 0)\cdot\nabla\Phi + \frac{\partial\Phi}{\partial t} \tag{27}$$

Taking into account that

$$\frac{\partial\Phi}{\partial x}(x, 0, \pm 0) = \pm\frac{\gamma(x)}{2} \tag{28}$$

the pressure difference between the airfoil's lower and upper surface is

$$\begin{aligned}\Delta p = p_l - p_u &= 2\rho\left[U_\infty\frac{\gamma(x)}{2} + \frac{\partial\Phi}{\partial t}\right] = \\ &= \rho U_\infty\gamma(x) + \rho\frac{\partial}{\partial t}\int_0^x \gamma(x)dx\end{aligned} \tag{29}$$

For the lumped-vortex method there is only one airfoil vortex and therefore the lift and drag per unit span are

$$\begin{cases} L' = \rho\left[U_\infty\Gamma(t) + \dfrac{\partial}{\partial t}\Gamma(t)c\right] \\ D' = \rho\left[w_W(x,t)\Gamma(t) + \dfrac{\partial}{\partial t}\Gamma(t)c\,\alpha\right] \end{cases} \tag{30}$$

One important parameter used in the description of unsteady aerodynamics und unsteady airfoil behavior is the reduced frequency, k, defined as $k = \omega\cdot c/(2V)$, where ω is the angular frequency, c is the chord of the airfoil and V is the flow velocity. According to the dimensional analysis, the resultant force, F, on the airfoil of chord c, can be written in functional form as $F/(\rho V^2 c^2) = f(Re, M, k)$. For $k = 0$ the flow is steady and for $0 \le k \le 0.05$ the flow can be considered quasi-steady, that is, unsteady effects are generally small. Flows with characteristic reduced frequencies above of 0.05 are considered unsteady [1].

The lift amplitude and phase of lift for pure angle of attack oscillations are presented in Fig. 7 and Fig. 8, where the significance of the apparent mass contribution to both the amplitude and phase can be appreciated.

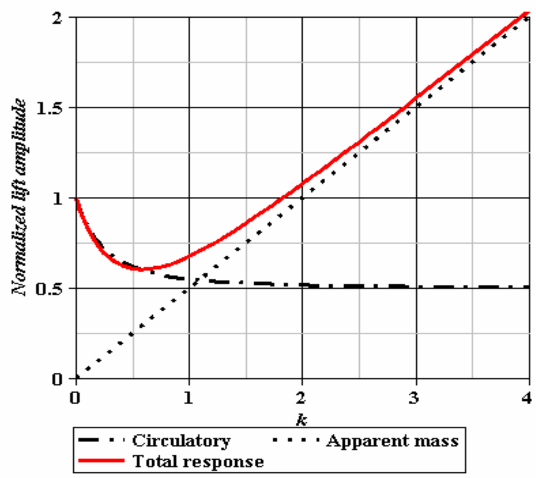

Fig. 7. Normalized lift amplitude

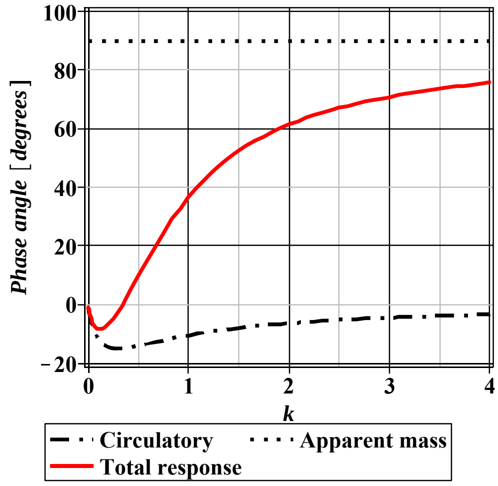

Fig. 8. Phase angle

At lower values of reduced frequency, the circulatory terms dominate the solution. At higher values of reduced frequency, the apparent mass forces dominate.

6. CONCLUSIONS

The lift at $t = 0_+$ is exactly half of the steady-state lift due to the acceleration portion of the lift that results from the change in the upwash, not due to the airfoil circulation.

The drag force has two components, one due to the wake-induced downwash which rotates the circulatory lift term by an induced angle w_w / U_∞ and other due to the fluid acceleration $\partial\Phi / \partial t$ which acts normal to the flat plate, and its contribution to the drag is the second lift term times α. One of the more difficult aspects of the unsteady problem is the modeling of the vortex wake's shape and strength, which depend on the time history of the motion.

The unsteady forces produced on a rotor blade arise primarily because of the vertical velocity between the wake disturbance and the airfoil surface. In linear theory, this is treated as an imposed unsteady upwash field, which must be used to satisfy the boundary conditions of flow tangency on the airfoil surface.

The airfoil can generate high lift as a result of a vortex that is shed at the leading edge at the instant of stall. The vortex travels back over the top of the airfoil carrying with it a low pressure wave that accounts for the very large lift coefficient.

REFERENCES

1. Gareth, D. P., *Helicopter Flight Dynamics,* AIAA Education Series (2007).
2. Gordon L., *Principles of Helicopter Aerodynamics,* Cambridge University Press (2007).
3. Katz, J., Plotkin, A., *Low Speed Aerodynamics,*Second Edition, Cambridge University Press, (2010).
4. Rotaru, C., Circiu, I., Boscoianu M. *Computational Methods for the Aerodynamic Design*, Review of the Air Force Academy, No 2(17), pp. 43-48 (2010).
5. Rotaru, C., Mihăilă M., Matei, P., Stefan, A., *Computing Nonlinear Characteristics of Helicopter Rotor Blades,* Review of the Airforce Academy, Vol. XI, No I(23), ISSN 2069-4733, pp.5-12 (2013).
6. Rotaru, C., Arghiropol A., *Maple soft solutions for nonlifting flows over arbitrary bodies,* Proceedings of the 3rd WSEAS international conference on FINITE DIFFERENCES - FINITE ELEMENTS - FINITE VOLUMES - BOUNDARY ELEMENTS, ISSN 1970-2769, pp. 270-274 (2010).
7. Maple soft documentation