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THE EQUATION OF DISPERSION AND THE DISPLACEMENT VECTOR IN THE SYMMETRIC CASE

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Abstract: *In this paper we study the propagation of the symmetric Lamb waves and we find the equation of dispersion and the equations of the displacement vector in the symmetric case.*

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1. INTRODUCTION

In this paper we consider an elastic, isotropic, continuous and homogeneous medium and we study the propagation of the Lamb waves through it. Using the results from [1], [2] and [3], we obtain the equation of dispersion and the equations of the displacement vector in the symmetric case.

2. PROBLEM FORMULATION

In the following we consider the normal guided Lamb waves. These waves appear in a plate of thickness $2h$ comparable with the wavelength, due to coupling between the components longitudinal L and the transverse components of the wave TV. Thus, two types of wave Lamb can be produced, but, in this parer, we study just the symmetric waves which are depicted in Figure 1, where for each side of the middle of the plate, the longitudinal components are equal and the transverse components are opposite.

We assume homogeneous and isotropic elastic plate bounded by two parallel planes located at a short distance $2h$, and we want to find the equation of dispersion and the equations of the displacement vector.

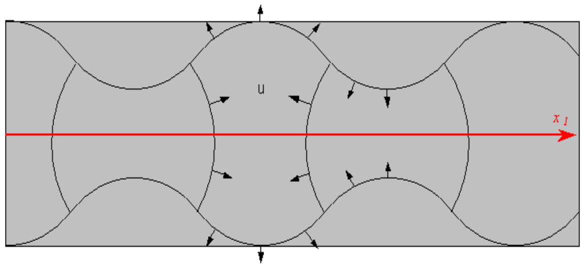

Figura 1: The symmetric Lamb waves

3. PROBLEM SOLUTION

In the article [1] we started from the equation of the displacement vector $\boldsymbol{u}$ for a material point

$$\mathbf{u} = \nabla\varphi + \nabla\times\psi \tag{1}$$

where φ is a scalar potential and ψ is a vectorial potential. In expression (1) the two potentials should verify the following two equations of wave

$$\nabla^2\varphi - \frac{1}{v_L^2}\frac{\partial^2\varphi}{\partial t^2} = 0, \tag{2}$$

where v_L is the phase velocity of the longitudinal waves and

$$\nabla^2\psi - \frac{1}{v_T^2}\frac{\partial^2\psi}{\partial t^2} = 0, \tag{3}$$

where v_T is the phase velocity of the longitudinal waves.

The scalar and the vectorial potentials are trigonometric functions of time *t*, with the same frequency ω. However, they can be expressed as follows, with the wave number *k*:

$$\begin{aligned} \phi &= \phi_0(x_2)e^{i(\omega t - kx_1)}, \\ \psi &= \psi_{0j}(x_2)e^{i(\omega t - kx_1)}, \quad j = 1,2,3. \end{aligned} \tag{4}$$

The equation of dispersion has the following relation:

$$\frac{\tan(qh+\alpha)}{\tan(ph+\alpha)} = -\frac{4k^2pq}{\left(q^2-k^2\right)^2} \tag{5}$$

with $\alpha = 0$ and $\alpha = \frac{\pi}{2}$, where the constants p and q are defined as follows:

$$p^2 = \frac{\omega^2}{v_L^2} - k^2, \qquad q^2 = \frac{\omega^2}{v_T^2} - k^2, \tag{6}$$

and the constant angle α can take the values 0 and $\frac{\pi}{2}$ depending on the type of symmetry of the wave.

Further, using the results from [3], we know that we have three regions with respect to the phase velocity. So, we write the equation of dispersion (5) and the equations of the displacement vector for each subdomain of wave number *k* taking into account the type of values that can be taken the constants *p* and *q*, for the case $\alpha = 0$ (the symmetric case) .

The first case is for $k < \frac{\omega}{v_L}$. We have *p* and *q* real numbers, so the dispersion equation is

$$\frac{\tan(qh+\alpha)}{\tan(ph+\alpha)} = -\frac{4k^2pq}{\left(q^2-k^2\right)^2}, \tag{7}$$

and the equations of the displacement vector are:

$$\begin{aligned} u_1 &= qA\left[\cos(qx_2) - \frac{2k^2}{k^2-q^2}\cdot\frac{\cos(qh)}{\cos(ph)}\cdot\cos(px_2)\right] \\ &\quad\cdot\cos(\omega t - kx_1), \\ u_2 &= -kA\left[\sin(qx_2) + \frac{2pq}{k^2-q^2}\frac{\cos(qh)}{\cos(ph)}\cdot\sin(px_2)\right] \\ &\quad\cdot\sin(\omega t - kx_1). \end{aligned} \tag{8}$$

The second case is for $\frac{\omega}{v_L} < k < \frac{\omega}{v_T}$. We have p an imaginar number, i.e. $p = ip$ and q a real number, so the dispersion equation has the form:

$$\frac{\tan(qh+\alpha)}{\tan(ph+\alpha)} = \frac{4k^2pq}{\left(q^2-k^2\right)^2}, \tag{9}$$

and the equations of the displacement vector are:

$$\begin{aligned} u_1 &= qA\left[\cos(qx_2) - \frac{2k^2}{k^2-q^2}\frac{\cos(qh)}{\cosh(ph)}\cosh(px_2)\right] \\ &\quad\cdot\cos(\omega t - kx_1), \\ u_2 &= -kA\left[\sin(qx_2) - \frac{2pq}{k^2-q^2}\frac{\cos(qh)}{\cosh(ph)}\sinh(px_2)\right] \\ &\quad\cdot\sin(\omega t - kx_1). \end{aligned} \tag{10}$$

The third case is for $\frac{\omega}{v_L} < \frac{\omega}{v_T} < k$. We have p and q imaginare numbers, i.e. $p = ip$ and $q = iq$, so the dispersion equation is

$$\frac{\tan(qh+\alpha)}{\tan(ph+\alpha)} = \frac{4k^2pq}{\left(q^2+k^2\right)^2} \tag{11}$$

and the equations of the displacement vector are the following form:

$$u_1 = qA\left[\cosh(qx_2) - \frac{2k^2}{k^2+q^2}\cdot\frac{\cosh(qh)}{\cosh(ph)}\cdot\right.$$
$$\left.\cdot\cosh(px_2)\right]\cos(\omega t - kx_1),$$

$$u_2 = -kA\left[\sinh(qx_2) + \frac{2pq}{k^2+q^2}\cdot\frac{\cosh(qh)}{\cosh(ph)}\cdot\right.$$
$$\left.\cdot\sinh(px_2)\right]\cos(\omega t - kx_1), \tag{12}$$

where A is a constant factor.

3. CONCLUSIONS

In this paper we use the symmetric Lamb waves in an elastic, isotropic, continuous and homogeneous medium and we find the equation of dispersion and the equations of the displacement vector for every subdomain of the wave number k in the symmetric case.

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