"HENRI COANDA"
AIR FORCE ACADEMY
ROMANIA

"GENERAL M.R. STEFANIK"
ARMED FORCES ACADEMY
SLOVAK REPUBLIC

INTERNATIONAL CONFERENCE of SCIENTIFIC PAPER
AFASES 2015
Brasov, 28-30 May 2015

PROJECTILE'S DRAG COEFFICIENT EVALUATION FOR SMALL FINITE DIFFERENCES OF HIS GEOMETRICAL DIMENSIONS USING ANALYTICAL METHODS

George Surdu*, **Ioan Vedinaş****, **Georgică Slămnoiu***, **Şomoiag Pamfil****

*Research Center for Navy, Constanta, Romania, **Military Technical Academy, Romania

Abstract: *In this paper is described an algorithm for projectile's drag coefficient evaluation for small finite differences of projectile's geometrical dimensions. The algorithm is presented in projectile design bibliography as projectile's shape index evaluation using empirical relations. The study is useful for engineers who work in research and ammunition design when is necessary to evaluate the preliminary projectile shape index and his implication on trajectory. In the same time based on projectile's shape index the drag coefficient is calculated using the reference law's Siacci, 1943 and 1930. The paper offer an evaluation of projectile's drag coefficient/shape index in accordance with the small finite differences of projectile's geometrical dimensions of ogive and tronconical part. This study can be easily implemented in a standalone application as a module of projectile's drag coefficient evaluation.*

Keywords: *drag coefficient, projectile's shape index, ammunition, aerodynamics, projectile, aerodynamic configuration, reference drag laws*

1. INTRODUCTION

Analytical methods are very useful for preliminary evaluation for aerodynamic parameters of projectiles and projectiles ballistic design. In this case is necessary a simple and expedite instrument to evaluate the influences for small differences of projectile's geometrical dimensions in drag coefficient evaluation or projectile's shape index. In addition, these small differences represent the accepted tolerance or accepted projectile's dimensions tolerances in ballistic design.

These types of studies offers to engineers or field test specialist of ammunitions an important standalone instrument which serves in: projectile geometrical modification evaluation, ballistic table evaluation, projectile's trajectory evaluation using drag coefficient modifications caused by small differences of projectile's dimensions.

This small differences/ tolerances of projectile's dimension are inherent and the only way is to accept them and evaluate the implication of them on projectile's aerodynamic coefficients, projectile's trajectory or projectile – target impact.

Some of these studies can represent a cheap and handy alternative for field-testing but cannot replace the experimental tests. Specialist for preliminary evaluation and implications can use these instruments.

This study presents projectile's shape index and drag coefficient evaluation using the method of the French researcher Hélie that has established a relation for the shape index for projectile based on his ogive dimensions.

In addition, this relation is corrected using the projectile shape index correction made some relations obtained based in aerodynamic tunnel experiments made in U.S.A.

This study put together these relations for projectile's shape index and also evaluate projectile's drag coefficient in accordance with the standard drag law's Siacci, 1943 and 1930.

This study also contains and exemplification of how small differences in projectile's geometry can affect the value of projectile's shape or drag coefficient.

The study has two main objectives: projectile's shape index and drag coefficient evaluation using the proposed analytical algorithm and the influence of projectile's small geometrical tolerances on projectile's shape index.

The evaluated drag coefficient is for Mach values between 0.1 and 4.0.

The projectile aerodynamic configuration is similar to the 30 mm projectile presented in Fig.1.

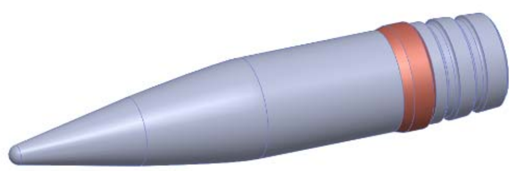

Fig. 1 Aerodynamic configuration of 30 mm caliber projectile used

The study uses some of the projectile's geometrical dimensions and his flight conditions.

2. MATHEMATICAL MODELS USED

The study has two main objectives as we mentioned before and for these objectives are reached by using simple empirical relations to evaluate the projectile's shape index and drag coefficient. In this case, the mathematical model for the analytical evaluation will be presented in the following.

The mathematical model [1, 2, 3] for drag coefficient estimation uses projectile's geometrical dimensions (Fig. 2).

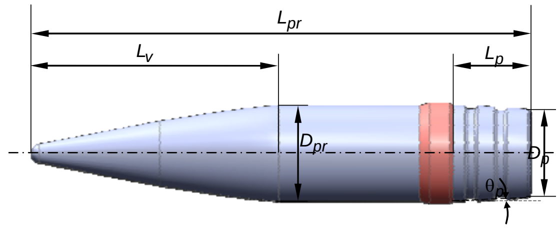

Fig. 2 Projectile's dimensions used

These dimensions are: L_{pr} – projectile's total length, L_v - ogive length, L_p - tronconical length, D_{pr} - transversal section diameter, D_p projectile back – side diameter, θ_p – angle for projectile's tronconical part.

For the algorithm of shape index estimation, we use the following relations [2,3]:

$$\lambda_v = \frac{L_v}{D_{pr}} \tag{1}$$

$$\gamma_p = D_{pr} - 2 \cdot L_p \cdot tg(\theta_p) \tag{2}$$

$$\gamma_{pr} = 1 - \frac{\gamma_p^2}{D_{pr}^2} \tag{3}$$

Where λ_v is ogive's relative length, γ_p represents a geometrical coefficient for tronconical part, γ_{pr} is a geometrical coefficient for projectile shape.

The preliminary projectile's shape index, taking into account the ogive's length, is estimated using relation [1]:

$$i_T = \frac{12 \cdot \lambda_v}{\sqrt{5} \cdot \left(4 \cdot \lambda_v^2 + 1\right)} \tag{4}$$

Using the above relation for projectile shape index evaluation we can obtain the final values for i_{TC} [1]:

$$i_{TC} = i_T \cdot \left[1 - \left(0.6 + \frac{\lambda_v}{10}\right) \cdot \gamma_{pr}\right] \tag{5}$$

valid for projectile's velocities less than 250 m/s,

$$i_{TC} = i_T \cdot \left[1 - \eta_{pr} \cdot \gamma_{pr}\right] \tag{6}$$

valid for projectile's velocities greater than 205 m/s and less and equal with 400 m/s and

$$\eta_{pr} = 0.6 + 0.167 \cdot \lambda_v - 0.0027 \cdot \lambda_v \cdot V_0 \tag{7}$$

For projectile's velocities greater than 400 m/s we use for shape index evaluation following relation [1]:

$$i_{TC} = i_T \cdot \left[1 - 0.6 \cdot \left(1 + \frac{\lambda_v}{10}\right) \cdot \gamma_{pr}\right] \quad (8)$$

Once we evaluate the projectile index , i_{TC}, we calculate drag coefficient for projectile using the reference drag coefficient law's Siacci, 1930 or 1943 based on projectile relative length. The relation for projectile's drag coefficient is:

$$C_D = i_{TC} \cdot C_{Dref} \quad (9)$$

In which C_D represents drag coefficient calculated and C_{Dref} represents the value of drag coefficient used as reference.

Based on relations (5) to (8) we can evaluate the influence of projectile's small geometrical tolerances on projectile's shape index using the small differences hypothesis [3,4,5,6] or in mathematical terms, the shape index functions differentiation as follows:

$$\delta i_{TC} = \Delta\lambda_v + \Delta L_p + \Delta\theta_p \quad (10)$$

In which:

$$\Delta\lambda_v = \frac{\partial i_{TC}}{\partial \lambda_v} \cdot \delta\lambda_v \quad (11)$$

$$\Delta L_p = \frac{\partial i_{TC}}{\partial L_p} \cdot \delta L_p \quad (12)$$

$$\Delta\theta_p = \frac{\partial i_{TC}}{\partial \theta_p} \cdot \delta\theta_p \quad (13)$$

$$\delta\lambda_v = \frac{\partial \lambda_v}{\partial L_v} \cdot \delta L_v + \frac{\partial \lambda_v}{\partial D_{pr}} \cdot \delta D_{pr} \quad (14)$$

$$\frac{\partial \lambda_v}{\partial L_v} = \frac{1}{D_{pr}} \quad (15)$$

$$\frac{\partial \lambda_v}{\partial D_{pr}} = -\frac{L_{pr}}{D_{pr}^2} \quad (16)$$

In which $\frac{\partial i_{TC}}{\partial \lambda_v}$, $\frac{\partial i_{TC}}{\partial L_p}$, $\frac{\partial i_{TC}}{\partial \theta_p}$, $\frac{\partial \lambda_v}{\partial L_v}$, $\frac{\partial \lambda_v}{\partial D_{pr}}$ are partial derivatives.

As an example of projectile's shape index variation caused by λ_v we have for index shape variation the following relations:

$$\delta i_{TC} = \frac{\partial i_{TC}}{\partial \lambda_v} \cdot \delta\lambda_v \quad (17)$$

where $\delta\lambda_v$ is from relation (14) and

$$\frac{\partial i_{TC}}{\partial \lambda_v} = \frac{i_{TC}}{\lambda_v \cdot \left(4 \cdot \lambda_v^2 + 1\right)} - \frac{6 \cdot i_{TC}}{100} \cdot \gamma_{pr} \quad (18)$$

Using relations (14), (17) and (18), we can evaluate the influence of projectile's ogive length on projectile's shape index and drag coefficient. In similar way can be evaluated, too, the influences on projectile shape index and drag coefficient caused by θ_p and L_p variations.

3. NUMERICAL RESULTS

Initial data used to make the calculations for projectile shape index and drag coefficient are presented in Table 1.

Table 1. Initial data for calculation

Parameter	Value
D_{pr} [mm]	30
L_{pr} [mm]	150.75
L_v [mm]	74.58
L_p [mm]	5.2
Mach number [-]	0.1 to 4.0
θ_p [deg]	15

Drag coefficient values obtained for this initial data are exposed in Table 2. These drag coefficient values are calculated using the mathematical model presented in chapter 2.

Table 2. Numerical results for shape index and drag coefficient

Mach number value	Shape index value	Drag coefficient after Siacci law	Drag coefficient after 1930 law
0.1	0.4408	0.1124	0.0815
0.2	0.4408	0.1124	0.0815
0.3	0.4408	0.1128	0.0815
0.4	0.4408	0.1128	0.0815
0.5	0.4408	0.1133	0.0815
0.6	0.4408	0.1142	0.0815
0.7	0.4408	0.1172	0.0815
0.8	0.5936	0.1692	0.1098
0.9	0.6146	0.2489	0.1278
1.0	0.6356	0.3471	0.2276
1.1	0.6567	0.4196	0.2364
1.2	0.4499	0.3104	0.1602
1.3	0.4499	0.3230	0.1566
1.4	0.4499	0.3289	0.1525
1.5	0.4499	0.3302	0.1476
1.6	0.4499	0.3302	0.1417
1.7	0.4499	0.3275	0.1363
1.8	0.4499	0.3230	0.1318
1.9	0.4499	0.3181	0.1287
2.0	0.4499	0.3122	0.1260
2.1	0.4499	0.3064	0.1233
2.2	0.4499	0.3001	0.1201
2.3	0.4499	0.2938	0.1161
2.4	0.4499	0.2875	0.1125
2.5	0.4499	0.2812	0.1093
2.6	0.4499	0.2749	0.1062
2.7	0.4499	0.2686	0.1030
2.8	0.4499	0.2632	0.1003
2.9	0.4499	0.2573	0.0976
3.0	0.4499	0.2515	0.0949
3.1	0.4499	0.2456	0.0922
3.2	0.4499	0.2411	0.0904
3.3	0.4499	0.2357	0.0882
3.4	0.4499	0.2312	0.0859
3.5	0.4499	0.2263	0.0841
3.6	0.4499	0.2218	0.0819
3.7	0.4499	0.2173	0.0801
3.8	0.4499	0.2132	0.0783
3.9	0.4499	0.2088	0.0769
4.0	0.4499	0.2052	0.0751

Table 3. Numerical results for ogive relative length

δL_v	δD_{pr}	$\lvert\delta\lambda_v\rvert$	$\lvert\delta\lambda_v\rvert$ caused by δL_v	$\lvert\delta\lambda_v\rvert$ caused by δD_{pr}
0.0007	0.0208	0.0017	0.0000	0.0017
0.0215	0.0341	0.0021	0.0007	0.0028
0.1078	0.0930	0.0041	0.0036	0.0077
0.1480	0.1136	0.0045	0.0049	0.0094
0.1638	0.1584	0.0077	0.0055	0.0131
0.2874	0.2683	0.0127	0.0096	0.0222
0.3080	0.3056	0.0151	0.0103	0.0253
0.3560	0.3483	0.0170	0.0119	0.0289
0.4140	0.3611	0.0161	0.0138	0.0299
0.4816	0.4127	0.0181	0.0161	0.0342
0.5528	0.4645	0.0201	0.0184	0.0385
0.6439	0.4765	0.0180	0.0215	0.0395
0.7238	0.4802	0.0157	0.0241	0.0398
0.7505	0.6683	0.0304	0.0250	0.0554
0.7759	0.7011	0.0322	0.0259	0.0581
0.8272	0.7212	0.0322	0.0276	0.0598
0.8538	0.7965	0.0375	0.0285	0.0660
0.9156	0.8968	0.0438	0.0305	0.0743
0.9628	0.9647	0.0478	0.0321	0.0799
0.9887	0.9705	0.0475	0.0330	0.0804

Calculated drag coefficient graph is presented in Figure 3. Results are obtained using the exposed analytical method.

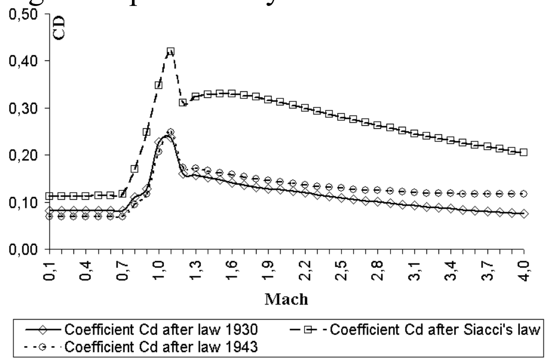

Fig. 3 Drag coefficient values vs. Mach using the algorithm presented

Results for projectile's ogive relative variation, $\delta\lambda_v$, caused by ogive length variation, δL_v, and projectile diameter variation, δD_{pr}, are presented in Table 3., Figure 4. to Figure 6.

Results for projectile's shape index variation with ogive length variations and ogive length and projectile's diameters variations are presented in Table 4., Figure 7. to Figure 9.

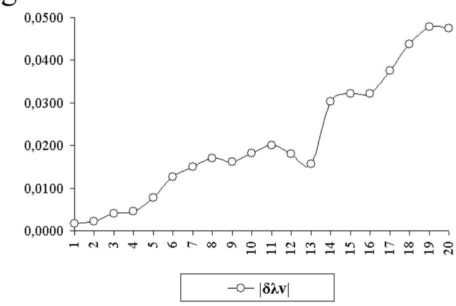

Fig. 4 $\lvert\delta\lambda_v\rvert$ for δL_v and δD_{pr} in the same time

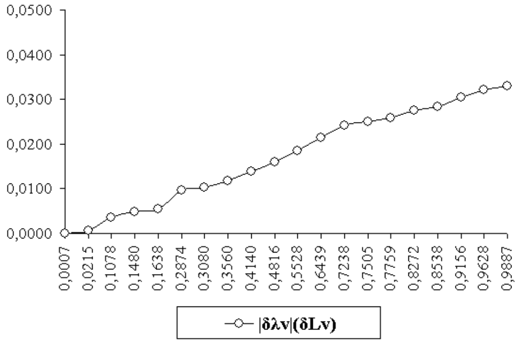

Fig. 5 $|\delta\lambda_v|$ only for δL_v

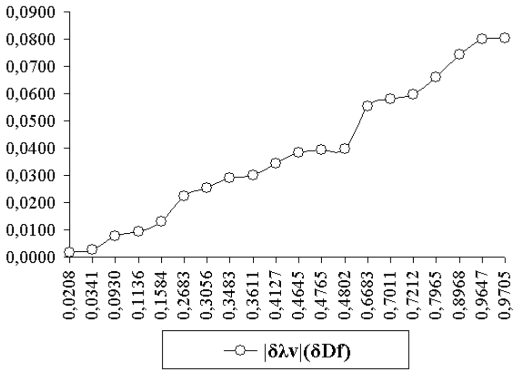

Fig. 6 $|\delta\lambda_v|$ only for δD_{pr}

Table 4. Numerical results for shape index

$\delta i_{TC}(\delta\lambda_v)$	$\lvert\delta i_{TC}\rvert(\delta L_v)$	$\lvert\delta i_{TC}\rvert(\delta D_{pr})$
0.00030	0.000004	0.00030
0.00037	0.00013	0.00050
0.00072	0.00063	0.00135
0.00079	0.00087	0.00165
0.00135	0.00096	0.00230
0.00222	0.00168	0.00390
0.00264	0.00180	0.00444
0.00298	0.00208	0.00506
0.00283	0.00242	0.00525
0.00318	0.00282	0.00600
0.00352	0.00323	0.00675
0.00316	0.00377	0.00693
0.00275	0.00423	0.00698
0.00533	0.00439	0.00972
0.00566	0.00454	0.01019
0.00565	0.00484	0.01049
0.00659	0.00499	0.01158
0.00768	0.00536	0.01304
0.00840	0.00563	0.01403
0.00833	0.00578	0.01411

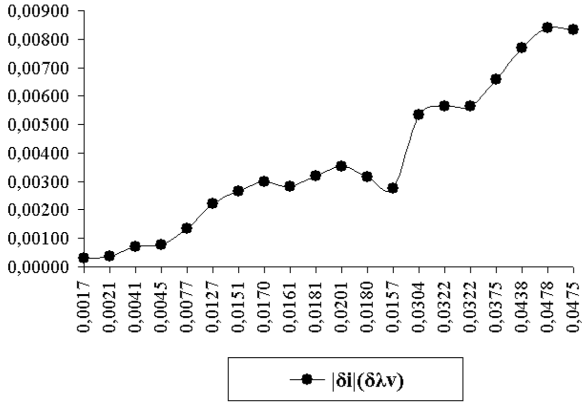

Fig. 7 $|\delta i_{TC}|$ only for $\delta\lambda_v$

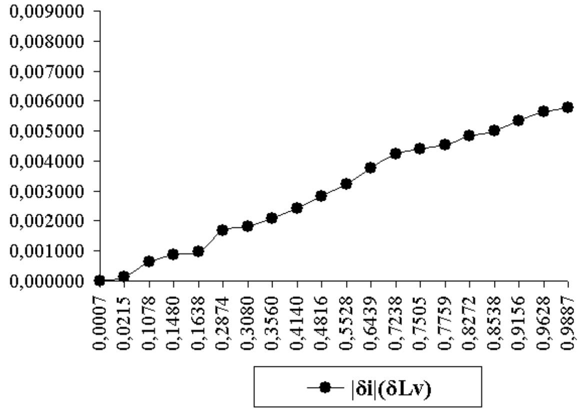

Fig. 8 $|\delta i_{TC}|$ only for δL_v

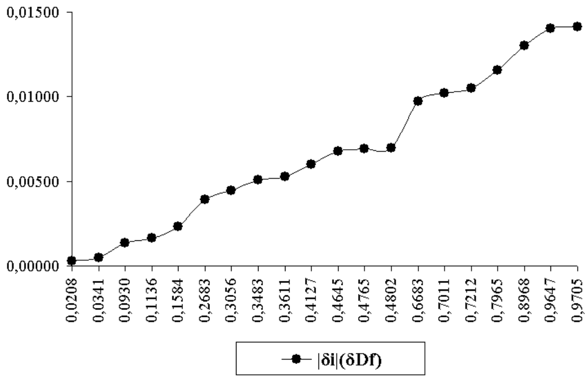

Fig. 9 $|\delta i_{TC}|$ only for δD_{pr}

As we can observe projectile's diameter variation introduce the highest variation of projectile's shape index.

We also can conclude that we can use for preliminary calculations this method to calculate the projectile's shape index and drag coefficient for aerodynamic configurations of projectiles. As a plus we can evaluate too drag coefficient variation caused by projectile's geometrical variations.

4. CONCLUSIONS & ACKNOWLEDGMENT

The drag coefficient was calculated, see Table 2 and Fig. 3 using the method presented in chapter 2.

On the other hand, the drag coefficient was calculated using the algorithm presented in chapter 2, and the results obtained for it were in accordance with preliminary design requirements.

In addition, reference [1] recommends this method to be used for preliminary projectiles design evaluations.

This kind of study can be used to implement the presented method as module for an exterior ballistic. The software module is a good ready-made instrument for evaluations instead of sheet preliminary calculations.

The usefulness of this type of study can be seen in teaching purposes, experimental testing and design of different type of products.

This paper has been financially supported within the project entitled "Horizon 2020 - Doctoral and Postdoctoral Studies: Promoting the National Interest through Excellence, Competitiveness and Responsibility in the Field of Romanian Fundamental and Applied Scientific Research", contract number POSDRU/159/1.5/S/140106. This project is co-financed by European Social Fund through Sectoral Operational Programme for Human Resources Development 2007-2013. Investing in people!

REFERENCES

1. Antonescu, I. *Bazele proiectării proiectilelor*, Editura Academiei Militare, Bucure□ti (1977).
2. Niță, M.M., Moraru, Fl., Patraulea, R., *Avioane şi rachete. Concepte de proiectare*, Editura Militară, Bucureşti (1985).
3. Moraru, Fl., *Elemente de teoria estimației cu aplicații în balistică*. Editura Academiei Tehnice Militare, Bucureşti (1979).
4. Surdu. G., Slămnoiu. G., Moldoveanu C., Ciuculin. A., Şomoiag P., *Metode statistice şi instrumente software realizate in – house pentru aplicații în balistică cu scopul determinării valorii adevărate a parametrilor determinați experimental.* Sisteme energetice şi Balistice. Bucureşti (2013).
5. Şomoiag, C.E. Moldoveanu, *Numerical Research on the Stability of Launching Devices During Firing,* Defence Technology Elsevier Journal, Volum 9, Issue 4, decembrie 2013, pag. 242-248, ISSN 2214-9147.
6. C.E. Moldoveanu, A. Giovannini, H.C. Boisson, *Turbulence receptivity of longitudinal vortex-dominated flows*, Lecture Notes in Computational Science and Engineering, Vol. 69: BAIL 2008 - Boundary and Interior Layer Proceedings of the International Conference on Boundary and Interior Layers - Computational and Asymptotic Methods, Limerick, July 2008, Springer Verlag, Berlin, pag. 227 – 236, ISBN: 978-3-642-00604-3, 2009.