# IDENTIFYING DATA AFFECTED BY ABERRANT ERRORS OBTAINED IN THE MANUFACTURING OF SPECIAL ALLOYS

**Ioan MILOSAN**

Transilvania University, Brasov, Romania (milosan@unitbv.ro)

DOI: 10.19062/2247-3173.2016.18.1.66

***Abstract:*** *The paper presents an example of calculation to identify data affected by aberrant errors and removing these data from the values of hardness (HB), strength ($R_m$) elongation (A) and impact strength (KCU) of Ni austempered ductile iron, by applying the $r_{max}/r_{min}$ and Romanowski tests. By studying all the data presented in this paper, following remarkable conclusion: both tests had identified as data affected by aberrant errors for the same values of the mechanical properties. In this way, the experimental results obtained are validated with a statistical confidence level of 95%.*

***Keywords:*** *data affected by aberrant errors, $r_{max}/r_{min}$ test, Romanowski test*

## 1. INTRODUCTION

By analyzing a number of experimental data it can happen that some abnormal values to be higher or lower than the rest of the results. These non-conforming values are often referred to as: aberrant errors, abnormal, anomalous, outliers, discordant observation, exceptions, surprises, peculiarities or contaminants in different application domains [1].

The question which arises is to use statistical tests to identify abnormalities and removing these data because of incorrect information about the study that they can induce.

The paper presents an example of calculation to identify data affected by aberrant errors and removing these data from the values of hardness (HB), strength (Rm), elongation (A) and impact strength (KCU) of Ni austempered ductile iron, by applying the $r_{max}/r_{min}$ and Romanowski tests.

## 2. IDENTIFYING DATA AFFECTED BY ABERRANT ERRORS

Identifying data affected by aberrant errors can be accomplished by applying the $r_{max}/r_{min}$ and Romanowski tests.

Solving the two tests is done in the following steps [2 - 4]:

(1) Grouping of data and determining the minimum ( $x_{min}$) and maximum ( $x_{max}$) of the experimental values;

a

b

a

b

(2a) Calculating the arithmetic average for the $r_{max}/r_{min}$ test, with the relationships [5]:

$$\bar{x} = \frac{1}{n}(x_1 + x_2 + .. + x_n) = \frac{1}{n}\sum_{i=1}^{n} x_i$$

(2b) Calculating the arithmetic average for the Romanowki test, with the relationships [5]:

$$\bar{x} = \frac{1}{n-1}(x_1 + x_2 + .. + x_n) = \frac{1}{n-1}\sum_{i=1}^{n-1} x_i$$

(3a) Calculating the dispersion, with the relationships [5]:

$$s^2 = \frac{1}{n}[(x_1 - \bar{x})^2 + .. + (x_n - \bar{x})^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

(3b) Calculating the dispersion, with the relationships [5]:

$$s^2 = \frac{1}{n-1}[(x_1 - \bar{x})^2 + .. + (x_n - \bar{x})^2 = \frac{1}{n-1}\sum_{i=1}^{n-1}(x_i - \bar{x})^2$$

(4) Calculating the average square diverting, with the relationships [5]:

$$s = \sqrt{s^2}$$

(5a) Calculating the $r_{max}/r_{min}$ test, with the relationships [5]:

$$r_{\max/\min} = \frac{\left|\bar{x} - x_{\max/\min}\right|}{s\sqrt{(n-1)/n}}$$

(5b) Calculating the Romanowski test, with the relationships [5]:

$$t_{calc} = \frac{\left|x_0 - \bar{x}\right|}{s\sqrt{n/(n-1)}}$$

(6) Determining the critical value of test the $r_{max}/r_{min}$ test, with the relationships: $r_{crit} = r_{\alpha;\nu}$

(6) Determining the critical value of test the Romanowki test, with the relationships: $t_{crit} = t_{\alpha;\nu}$

(7) Comparing the $r_{max}/r_{min}$ calculated value with the critical value $r_{crit}$ and analyzing the results and the conclusions of this test

(7) Comparing the $t$ calculated value with the critical value $t_{crit}$ and analyzing the results and the conclusions of this test

where:

n = the number of experimental determinations;

$\bar{x}$ = the arithmetic average of the experimental data. For Romanowski test, the average value $\bar{x}$ is calculated without considering the value $x_0$ which is analyzed;

$s^2$ = the dispersion of the experimental data;

s = the average square diverting of the experimental data;

$x_0$ = the liable data to be affected by aberrant errors for the Romanowski test;

$r_{crit}$ = the critical value for the $r_{max}/r_{min}$ test [3];

$t_{crit}$ = the critical value for the *t* test is the tabular value on the basis of the number of values in the series of data and confidence interval [5];

α = the coefficient of statistical confidence level, α = 0.95 [6];

ν = the number of degrees of freedom, $\nu = n-2$ for the $r_{max}/r_{min}$ test and $\nu = n-1$ for the Romanowski test;

Comparing the $r_{max}/r_{min}$ calculated value with the critical value $r_{crit}$ and analyzing the results for the $r_{max}/r_{mi}$ test, two situations can occur:

a) If $r_{max}/r_{min} < r_{crit}$ the values of the minimum ( $x_{min}$) and maximum ( $x_{max}$) of the experimental values are normal for our experimental research;

b) If $r_{max}/r_{min} > r_{crit}$ the values of the minimum ( $x_{min}$) and maximum ( $x_{max}$) of the experimental values are identified as data affected by aberrant errors for our experimental research;

Comparing the *t* calculated value with the critical value $t_{crit}$ and analyzing the results for the Romanowski test, two situations can occur:

a) If $t < t_{crit}$ the values of the liable data to be affected by aberrant errors ($x_0$) of the experimental values are normal for our experimental research;

b) If $t > t_{crit}$ the values of liable data to be affected by aberrant errors ($x_0$) of the experimental values;

In both cases when determining abnormal after removing them, the calculations will made using new parameters required analysis.

## 3. EXPERIMENTAL INVESTIGATION

### 3.1. Materials

The studied materials, was a Ni-Cr cast iron with the following composition (% in weight): 3.75 %C; 2.14 %Si; 0.40 %Mn; 0.014%P; 0.004 %S; 0.05 %Mg and 0.48 %Ni.

This cast iron was made in an induction furnace. Nodular changes were obtained with the "In mold" method, with the help of prealloy FeSiMg with 10-16% Mg, added into the reaction chamber in a proportion of 1.1% of the treated cast iron.

This materials were subjected to a heat treatment whose parameters have been:

- the austenizing temperature, $T_A$ = 830 [°C];
- the maintained time at austenizing temperature, $\tau_A$ = 60 [min];
- the temperature at isothermal level, $T_{iz}$ = 300[°C] for lot A and 400[°C] for lot B;
- the maintained time at the isothermal level, $\tau_{iz}$ = 10, 20, 30, 40, 50 and 60 [min], corresponding to the first stage for obtaining bainitic structure.

All these experimental specimens, were performed at isothermal maintenance in salt-bath (55% KNO3+45% NaNO3), being the cooling after the isothermal maintenance was done in air.

From this material, 24 typical test specimens were done: 12 typical test specimens for hardness (HB), strength ($R_m$) and elongation (A) and 12 typical test specimens for impact strength (KCU)

### 3.2. Results

The values of the mechanical results are presented in tables 1.

Table 1. Data Analysis of HB, $R_m$ KCU and A values

| Lot | $T_A$ [°C] | $\tau_A$ [min] | $T_{iz}$ [°C] | $\tau_{iz}$ [min] | HB | $R_m$ [N /mm $^2$ ] | KCU [J/cm $^2$ ] | A [%] |
|---|---|---|---|---|---|---|---|---|
| A | 830 | 60 | 300 | 30 | 451 | 1400 | 54 | 2 |
| | | | | 60 | 426 | 1370 | 58 | 2,3 |
| | | | | 90 | 415 | 1350 | 63 | 2,5 |
| | | | | 120 | 408 | 1300 | 65 | 2,7 |
| | | | | 150 | 390 | 1190 | 63 | 2,6 |
| | | | | 180 | 383 | 1130 | 64 | 2,6 |
| B | | | 400 | 30 | 375 | 1020 | 73 | 4,2 |
| | | | | 60 | 321 | 930 | 78 | 4,8 |
| | | | | 90 | 306 | 890 | 77 | 5,3 |
| | | | | 120 | 302 | 860 | 79 | 5,6 |
| | | | | 150 | 298 | 850 | 77 | 5,4 |
| | | | | 180 | 292 | 815 | 76 | 5,3 |

It can be certainly observed a normal evolution of the values for mechanical characteristics [7]:

-when maintaining time at the isothermal level is growing then $R_m$ and HB are decreasing and A with KCU are increasing;

-when maintaining time at the same temperature of the isothermal level is increasing than $R_m$ and HB are decreasing, A and KCU are increasing;

- less maintaining time for the isothermal variation provides higher values of HB and $R_m$ but lower of A and KCU;

This evolution of the mechanical properties is determined by the structural changes reported to the parameters of the heat treating. This evolution of the mechanical properties is determining by the structural constituents for each heat treatment.

In the case of lots A structure can be constituted of inferior bainite, residual austenite and martensite .These constituents are determining high values for $R_m$ and HB, and less high for A and KCU.

This can be explained by the time of the isothermal level maintenance, followed by air cooling at the room temperature, is increasing the proportion of martensite, a constituent which is determining higher values for $R_m$ and HB and lower for A and KCU in the structure of the lots.

In the case of lots B structure can be constituted of superior bainite, residual austenite and the martensite will disappear. These constituents are determining high values for A and KCU, and less high for $R_m$ and HB [7].

## 4. IDENTIFYING DATA AFFECTED BY ABERRANT ERRORS

Experimental data of HB, $R_m$ KCU and A presented in table 1 are processed by one of the 2 tests the $r_{max}/r_{min}$ and Romanowski tests.

### 4.1. Data processing using the $r_{max}/r_{min}$ test

The $r_{max}/r_{min}$ test was applied following calculation steps above. Data resulting are presented in table 2. It was selected the coefficient of statistical confidence level, α = 0.95 (the default is 5% or 0.05). According [ 3] for an accuracy of 95%, the value of $r_{crit}$ = 1,996

Table 2. $r_{max}/r_{min}$ test for data sets

| Lot | Properties | x | Measured values | $\bar{x}$ | s | $r_{calc}$ | $r_{crit}$ | Values remains Yes/No |
|---|---|---|---|---|---|---|---|---|
| | HB | $x_{max}$ | 451 | 412.167 | 22.608 | 1.875 | | Yes |
| | | $x_{min}$ | 383 | | | 1.408 | | Yes |
| | $R_m$ | $x_{max}$ | 1400 | 1290 | 98.169 | 1.227 | | Yes |
| | | $x_{min}$ | 1130 | | | 1.785 | | Yes |
| A | KCU | $x_{max}$ | 65 | 61.167 | 3.891 | 1.079 | | Yes |
| | | $x_{min}$ | 54 | | | 2.017 | | *No* |
| | A | $x_{max}$ | 2.7 | 2.467 | 0.249 | 1.026 | | Yes |
| | | $x_{min}$ | 2 | | | 2.057 | | *No* |
| o | HB | $x_{max}$ | 375 | 315.667 | 27.992 | 2.322 | 1.996 | *No* |
| | | $x_{min}$ | 292 | | | 0.926 | | Yes |
| | $R_m$ | $x_{max}$ | 1020 | 894.167 | 66.484 | 2.070 | | *No* |
| | | $x_{min}$ | 815 | | | 1.304 | | Yes |
| B | KCU | $x_{max}$ | 79 | 76.667 | 1.886 | 1.330 | | Yes |
| | | $x_{min}$ | 73 | | | 2.090 | | *No* |
| | A | $x_{max}$ | 5.6 | 5.1 | 0.469 | 1.168 | | Yes |
| | | $x_{min}$ | 4.2 | | | 2.102 | | *No* |

Comparing the $r_{max}/r_{min}$ calculated value, $r_{calc}$ with the critical value $r_{crit}$ and analyzing the results for the $r_{max}/r_{mi}$ test, two situations can occur:

a) If $r_{max}/r_{min} < r_{crit}$ the values of the minimum ( $x_{min}$) and maximum ( $x_{max}$) of the experimental values are normal for our experimental research, in this case it noted with a *Yes;*

b) If $r_{max}/r_{min} > r_{crit}$ the values of the minimum ( $x_{min}$) and maximum ( $x_{max}$) of the experimental values are identified as data affected by aberrant errors for our experimental research, in this case it noted with a *No;*

By studying all the data presented in this table, following a general remarkable conclusions: all values are accepted as correct data (Yes) with exception the following values noted with No, which have been identified as data affected by aberrant errors :

- the value of $x_{min}$ =54, from the impact strength (KCU) data of lot A, $r_{calc}$ = 2.017, $r_{crit}$ = 1.996 so $r_{calc} > r_{crit}$ ;
- the value of $x_{min}$ = 2, from the elongation (A) data of lot A, $r_{calc}$ = 2.057, $r_{crit}$ = 1.996 so $r_{calc} > r_{crit}$ ;
- the value of $x_{max}$ = 375, from the hardness (HB) data of lot B, $r_{calc}$ = 2.322, $r_{crit}$ = 1.996 so $r_{calc} > r_{crit}$ ;
- the value of $x_{max}$ = 1020, from the strength ($R_m$) data of lot B, $r_{calc}$ = 2.070, $r_{crit}$ = 1.996 so $r_{calc} > r_{crit}$ ;
- the value of $x_{min}$ = 73, from the impact strength (KCU) data of lot B, $r_{calc}$ = 2.090, $r_{crit}$ = 1.996 so $r_{calc} > r_{crit}$ ;

- the value of $x_{min}$ = 4.2, from the elongation data of lot B, $r_{calc}$ = 2.102, $r_{crit}$ = 1.996 so $r_{calc} > r_{crit}$ ;

For this six experiments it was eliminated the value affected by aberrant errors and recovery the test without these values, until they finally obtain a value that confirms initial relationship: $r_{max}/r_{min} < r_{crit}$ . These new experimental value is normal for our experimental research and in this case it noted with a *Yes* for the column of the table representing values remains.

### 4.2. Data processing using the Romanowski test

The Romanowski test was applied following calculation steps above. Data resulting are presented in table 3. It was selected the coefficient of statistical confidence level, α = 0.95 (the default is 5% or 0.05). According [2, 5] for an accuracy of 95%, the value of $t_{crit}$ = 1,92.

The values of the Romanowski test for the data sets are presented in tables 3.

Table 3. Romanowski test for data sets

| Lot | Properties | x | Measured values | $\bar{x}$ | s | $t_{calc}$ | $t_{crit}$ | Values remains Yes/No |
|---|---|---|---|---|---|---|---|---|
| A | HB | $x_{max}$ | 451 | 404.4 | 24.375 | 1.746 | 1.92 | Yes |
| | | $x_{min}$ | 383 | 418 | 20.890 | 1.768 | | Yes |
| | $R_m$ | $x_{max}$ | 1400 | 1268 | 93.038 | 1.290 | | Yes |
| | | $x_{min}$ | 1130 | 1322 | 73.593 | 2.383 | | Yes |
| | KCU | $x_{max}$ | 65 | 60.4 | 3.826 | 1.098 | | Yes |
| | | $x_{min}$ | 54 | 62.6 | 2.417 | 3.250 | | *No* |
| | A | $x_{max}$ | 2.7 | 2.40 | 0.248 | 0.111 | | Yes |
| | | $x_{min}$ | 2 | 2.54 | 0.136 | 3.624 | | *No* |
| B | HB | $x_{max}$ | 375 | 303.8 | 9.765 | 6.659 | | *No* |
| | | $x_{min}$ | 292 | 320.4 | 28387 | 0.914 | | Yes |
| | $R_m$ | $x_{max}$ | 1020 | 869 | 38.781 | 3.556 | | *No* |
| | | $x_{min}$ | 815 | 910 | 61.664 | 1.407 | | Yes |
| | KCU | $x_{max}$ | 79 | 76.2 | 1.720 | 1.487 | | Yes |
| | | $x_{min}$ | 73 | 77.4 | 1.019 | 3.942 | | *No* |
| | A | $x_{max}$ | 5.6 | 5 | 0.452 | 1.212 | | Yes |
| | | $x_{min}$ | 4.2 | 5.28 | 0264 | 3.737 | | *No* |

Comparing the t calculated value, $t_{calc}$ with the critical value $t_{crit}$ and analyzing the results for the Romanowski test, two situations can occur:

a) If $t_{calc} < t_{crit}$ the values of the minimum ( $x_{min}$) and maximum ( $x_{max}$) of the experimental values are normal for our experimental research, in this case it noted with a *Yes;*

b) If $t_{calc} > t_{crit}$ the values of the minimum ( $x_{min}$) and maximum ( $x_{max}$) of the experimental values are identified as data affected by aberrant errors for our experimental research, in this case it noted with a *No;*

By studying all the data presented in this table, following a general remarkable conclusions: all values are accepted as correct data (Yes) with exception the following values noted with No, which have been identified as data affected by aberrant errors :

- the value of $x_{min}$ =54, from the impact strength (KCU) data of lot A, $t_{calc}$ = 3.250, $r_{crit}$= 1.92 so $t_{calc} > t_{crit}$;

- the value of $x_{min} = 2$, from the elongation (A) data of lot A, $r_{calc} = 2.057$, $t_{crit} = 1.92$ so $t_{calc} > t_{crit}$;

- the value of $x_{max} = 375$, from the hardness (HB) data of lot B, $t_{calc} = 3.624$, $r_{crit} = 1.92$ so $t_{calc} > t_{crit}$;

- the value of $x_{max} = 1020$, from the strength ($R_m$) data of lot B, $r_{calc} = 6.659$, $t_{crit} = 1.92$ so $t_{calc} > t_{crit}$;

- the value of $x_{min} = 73$, from the impact strength (KCU) data of lot B, $r_{calc} = 3.942$, $t_{crit} = 1.92$ so $t_{calc} > t_{crit}$;

- the value of $x_{min} = 4.2$, from the elongation data of lot B, $r_{calc} = 3.737$, $t_{crit} = 1.92$ so $t_{calc} > t_{crit}$;

For this six experiments it was eliminated the value affected by aberrant errors and recovery the test without these values, until they finally obtain a value that confirms initial relationship: $t_{calc} < t_{crit}$. These new experimental value is normal for our experimental research and in this case it noted with a *Yes* for the column of the table representing values remains.

## CONCLUSIONS

By studying all the data presented in this paper following remarkable conclusions:

(a) When maintaining time at the isothermal level is growing then $R_m$ and HB are decreasing and A with KCU are increasing;

(b) When maintaining time at the same temperature of the isothermal level is increasing than $R_m$ and HB are decreasing , A and KCU are increasing;

(c) Less maintaining time for the isothermal variation provides higher values of HB and $R_m$ but lower of A and KCU;

(d) The evolution of the mechanical properties is determined by the structural changes reported to the parameters of the heat treating and this evolution of the mechanical properties is determining by the structural constituents for each heat treatment;

(e) In the case of lots A, structure can be constituted of inferior bainite, residual austenite and martensite .These constituents are determining high values for $R_m$ and HB, and less high for A and KCU;

(f) In the case of lots B, structure can be constituted of superior bainite, residual austenite and the martensite will disappear. These constituents are determining high values for A and KCU, and less high for $R_m$ and HB;

(g) By using the two statistical tests (the $r_{max}/r_{min}$ and Romanowski tests) were removed normal results of experimental research;

(h) By studying all the data presented in the table 2 and 3, following a general remarkable conclusions: all experimental data as correct were noted with *Yes* and all experimental data as aberrant errors resulting from application, were noted with *No*;

(i) Both tests had identified as data affected by aberrant errors for the same values of the mechanical properties;

(j) Both tests will be recalculated until are done the conditions for acceptance.

## REFERENCES

[1] M. R. Costescu, Identifying Data Affected by Aberrant Errors. Applied Program, *Economic Informatics*, vol. 12, no. 1, pp. 52 - 22, 2008;

[2] M. Banica and N. Medan, Statistical analysis of the experimental data obtained in studying of water jet cleaning, *Academic Journal of Manufacturing Engineering*, vol. 13, no. 1, pp. 6-10, 2015;

[3] D. Taloi, *Optimization of metallurgical processes*, E.D.P. Bucharest, 1983;

[4] T. Laurian and R. F. Mirică, *Paper 3-Eliminating aberrant values*, 2014. Available at http://www.omtr.pub.ro/didactic/fsm/lab/fsm03.pdf, accessed on 28 April 2016;
[5] I. Tanasescu, *Statistical control of processes and products*, E.D.P. Bucharest, 1987;
[6] M. Bulgaru and L. Bolboaca, *Quality engineering. Quality management, statistics and control measurements in 3D,* Alma Mater, Cluj-Napoca, 2001;
[7] J. M. Chou, M. H. Hon. and J. L. Lee, The austenite transformation in ferritic ductile cast iron, *Materials Science and Engineering A,* vol. 158, no. 2, pp. 241-249. 1992.