AN H_∞ DESIGN METHOD FOR THE PITCH ATTITUDE HOLD AUTOPILOT OF A FLYING UAV

Adrian-Mihail STOICA, Petrisor-Valentin PARVU, Costin ENE

"Politehnica" University of Bucharest, Romania (adrian.stoica@upb.ro, parvupv@gmail.com, ene.costin27@gmail.com)

DOI: 10.19062/2247-3173.2016.18.1.24

Abstract: *The paper presents an H_∞ loop shaping method for the design of the automatic flight control system of an unmanned air vehicle's (UAV's) longitudinal dynamics. The design objectives include robust stabilization with respect to modeling uncertainties, pitch angle tracking of an ideal model and reduced sensitivity with respect to low frequency measurement errors. The design is illustrate by a case study for a flying wing UAV.*

Keywords: *UAV, pitch attitude hold autopilot, loop shaping, robust stability, ideal model tracking*

1. INTRODUCTION

The applications based on Unmanned Air Vehicle's (UAV's) received much attention over the last decades. Among them, the flying wing configuration of the UAVs was intensively analyzed due to their advantages concerning the reduced drag and the fuel consumption. A disadvantage of this configuration with respect to the classical one is its instability which fact requires an automatic control system able to stabilize the aircraft and to provide acceptable maneuverability qualities. The design problem of such a system has been previously considered in the control literature (see e.g. [1], [2], [3]). The proposed design methods include both classical techniques based for instance on the well-known PID (Proportional Integral Derivative) control laws, optimal approaches based on the linear quadratic problem, the H_∞ norm minimization and nonlinear methods including nonlinear inversion, etc ([4], [5], [6], [7], [8]). The methods mentioned above have advantages and drawbacks; the main difficulty in choosing the control design method is to ensure a trade-off between the complexity of the automatic flight control system and its performances. The actual applications require a wide spectrum of performances including not only the stabilization of the aircraft but also robustness with respect to modeling uncertainties and flying conditions changes, time response performances and reduced sensitivity to disturbances and measurement errors.

The aim of this paper is to present a design methodology for the Pitch Attitude Hold (PAH) autopilot of a flying wing configuration. The design procedure is illustrated and investigated for the Hirrus UAV designed and manufactured by a private Romanian company in collaboration with academics from Faculty of Aerospace Engineering of University "Politehnica" of Bucharest. The design approach is based on a modified version of the so-called two degrees of freedom (2 DOF) H_∞ loop-shaping able to accomplish simultaneously several objectives as robustness stability, model tracking and disturbances attenuation requirements. The original 2 DOF H_∞ method presented in [9] have been previously used for the autopilot design of the aircraft longitudinal and lateral dynamics (see [10], [11], [12]). In the present paper the 2 DOF H_∞ loop-shaping procedure is used for the design of the PAH autopilot of UAV with flying wing configuration. The paper is organized as follows: in the second section, the design objectives and the design model of the UAV are presented. The design procedure of the PAH autopilot is presented in the third section. The numerical results obtained in a case study for the Hirrus UAV are presented and analyzed in the fourth section. The paper ends with some final remarks.

2. DESIGN MODELS AND AUTOPILOT SYNTHESIS OBJECTIVES

The Hirrus platform illustrated in Fig. 1 has a flying wing configuration with the wingspan 3.2 m, length 1.2 m, maximum takeoff weight 6.5 kg, cruising speed of 80 km/h and with a maximum payload of 1 kg.

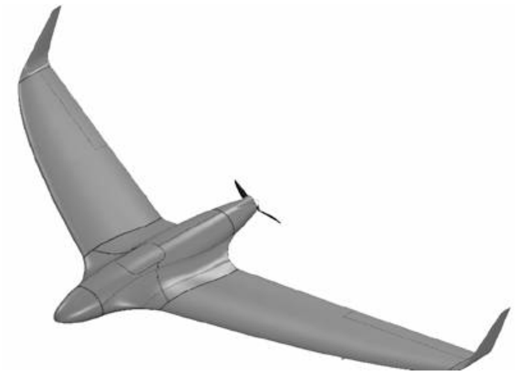

FIG.1. Hirrus UAV [13]

The longitudinal dynamics of the UAV is approximated by the following linearized nominal model:

$$\dot{x}(t) = Ax(t) + B\delta_e(t) \tag{1}$$

where the state vector $x(t)$ includes the perturbational values of the longitudinal speed u, of the angle of attack a, of the pitch rate q and of the pitch angle θ, respectively, with respect to their trimming conditions. The control input δ_e is the elevator deflection. The matrices arising in the above model corresponding a to the nominal flight conditions $V = 15\,\text{m/sec}$ and $h = 10\,\text{m}$ are

$$A = \begin{bmatrix} -1.3374e-02 & 1.3589e+00 & 0 & -9.07036e+00 \\ 1.2592e-01 & -1.1151e+01 & -5.0437e+01 & -1.4134e+00 \\ -7.2028e+00 & 1.3765e+03 & -3.5594+01 & 2.6574e-01 \\ 0 & 0 & 1 & 0 \end{bmatrix}, B = \begin{bmatrix} -6.4712e-03 \\ 3.6575e-02 \\ -7.7492e+00 \\ 0 \end{bmatrix} \tag{2}$$

The eigenvalues of the state matrix from the above system are $\{$-2.3362e+01 ± 2.6320e+02i;-1.6714e-02 ± 1.1328e-01i$\}$ showing that the UAV's phugoid poles are very close from the imaginary axis of the complex plane. This fact shows that in the absence of an automatic flight control system the time responses of the pitch angle are very slow. The measured outputs considered in this application are $y = [\alpha, q, \theta]^T$. The design objectives for the PAH autopilot are the following

(DO1) Robust stabilization of the longitudinal dynamics;

(DO2) Zero steady state value for the tracking error θ_{com}-$\theta(t)$ for piecewise constant values of the commanded pitch angle θ_{com} ;

(DO3) Shorter time responses of the pitch angle at constant commands;

(DO4) Reduced sensitivity with respect to low frequency measurement errors and robust stability with respect to modeling uncertainties.

3. THE DESIGN APPROACH OF THE PAH AUTOPILOT

In order to accomplish the design objective (DO2) defined in the previous section one introduces a *reference model* having the transfer function

$$H_m(s) = \frac{\omega_m^2}{s^2 + 2\xi_m \omega_m s + \omega_m^2} \tag{2}$$

where the natural frequency ω_m and the damping factor ξ_m are chosen such that the settling time is much shorter than in the case of uncontrolled phugoid. Thus, for $H_m(s) = 10/(s^2 + 0.8s + 10)$ the settling time is $t_s = 5.33\sec$ and the damping ratio is $\xi_m = 0.12$. The control configuration used for the PAH autopilot is presented in Fig. 2.

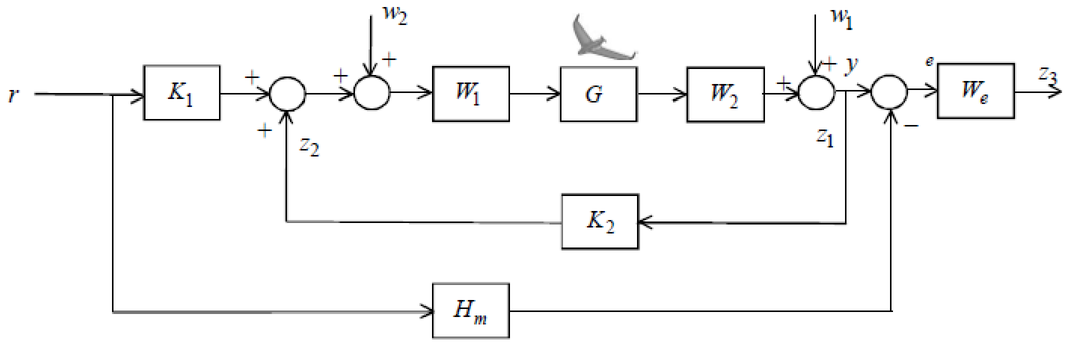

FIG. 2. PAH Autopilot configuration with 2 DOF

The two components K_1 and K_2 of the PAH autopilot are designed in order to accomplish the design objectives (DO1)¸ (DO4). In the above figure, W_1 is a stable dynamic weighting and $W_2 > 0$ is a positive weighting, chosen in order to provide the sensitivity reduction to measurements errors and robustness stability performances. The dynamic weighting $W_e(s)$ as a low-pass filter aiming to improve the tracking performance at low frequency commands for θ_{com} (see e.g. [15]). The control system $K = [K_1 \quad K_2]$ are determined such that it stabilizes the configuration from Fig. 2 and minimizes the H_∞ norm of the mapping $u_1 \to y_1$, where

$$u_1 = \begin{bmatrix} \theta_{com} \\ w_1 \\ w_2 \end{bmatrix} \quad \text{and} \quad y_1 = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}.$$

Reducing the H_∞ norm of the dependence from θ_{com} to z_3 implies to accomplish the tracking performances of the reference model output. In fact the control configuration from Fig. 2 may be regarded as the following H_∞ - norm minimization problem.

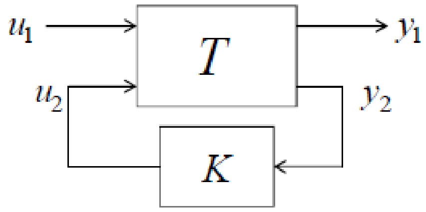

FIG. 3. The $H_¥$ -norm minimization problem

where T denotes the so-called *generalized system* obtained from the Fig.2 taking into account that u_2 and y_2 coincide with the control input and with the measurement vector, respectively, namely $u_2 = \delta_e$ and $y_2 = [\alpha, q, \theta]^T$. The generalized system T has the state-space realization of the following form

$$\begin{aligned}\dot{x} &= Ax + B_1u_1 + B_2u_2 \\ y_1 &= C_1x + D_{11}u_1 + D_{12}u_2 \\ y_2 &= C_2x + D_{21}u_1 + D_{22}u_2\end{aligned} \tag{3}$$

Denoting by (A_m, B_m, C_m) a minimal state-space realization of the ideal model H_m given in (3), by (A_s, B_s, C_s) a minimal realization of the *shaped system* $G_s = W_2GW_1$ and by (A_w, B_w, C_w) a realization of the dynamic weighting W_3 it follows that

$$A = \begin{bmatrix} A_s & 0 & 0 \\ 0 & A_m & 0 \\ B_wT_rC_s & -B_wC_m & A_w \end{bmatrix}, B_1 = \begin{bmatrix} 0 & -H \\ B_m & 0 \\ 0 & B_wT_r \end{bmatrix}, B_2 = \begin{bmatrix} B_s \\ 0 \\ 0 \end{bmatrix}, C_1 = \begin{bmatrix} C_s & 0 & 0 \\ 0 & 0 & 0 \\ D_wT_rC_s & -D_wC_m & C_w \end{bmatrix}$$

$$C_2 = \begin{bmatrix} 0 & 0 & 0 \\ C_s & 0 & 0 \end{bmatrix},\ D_{11} = \begin{bmatrix} 0 & I \\ 0 & 0 \\ 0 & D_wT_r \end{bmatrix},\ D_{12} = \begin{bmatrix} 0 \\ W_u \\ 0 \end{bmatrix},\ D_{21} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix},$$

where $T_r = [0 \quad 0 \quad 1]$ selects the regulated output θ from the measurement vector y, namely $T_ry = \theta$ and where $H = -ZC_s$, Z standing for the stabilizing solution of the Riccati equation ([15])

$$A_sZ + ZA_s^T - ZC_s^TC_sZ + B_sB_s^T = 0.$$

The positive weighting W_u has been introduced in the matrix D_{12}. It weights the control variable δ_e in order to reduce its amplitude for the saturation avoidance. Solving the above H_∞ norm minimization problem one obtains the 2 DOF controller $K = [K_1 \quad K_2]$, representing for the PAH autopilot with the structure shown in Fig. 4.

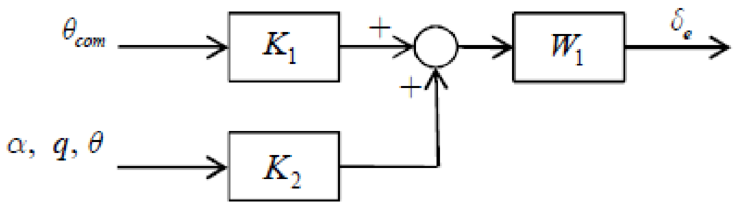

FIG.4. The 2 DOF autopilot

4. CASE STUDY FOR THE HIRRUS UAV

Before illustrating the design methodology presented above one notices from Fig. 5 that in the absence of the autopilot, the time response at step commands for the pitch angle is large due to the very small damping ratio of the phugoid.

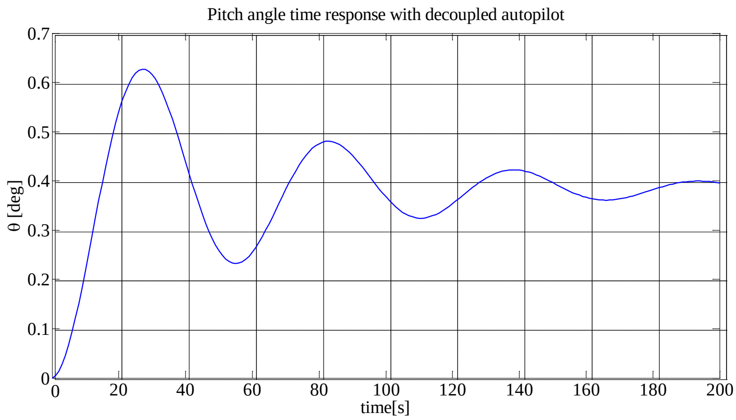

FIG.5. Pitch angle time response to unit step command

In order to improve the flying qualities of the UAV we used the design methodology presented in the previous section. One considered the linear model (1), (2) of the Hirrus UAV and then the H_∞ norm minimization problem for the system (4) have been solved using the following values for the weighting coefficients

$$W_1(s) = \frac{10^5}{100s+1},\ W_2 = I_3 \text{ and } W_e(s) = \frac{10^6(0.01s+1)}{100s+1}.$$

Two cases have been considered, corresponding to two different values of the weighting W_u penalizing the control variable δ_e, namely $W_u = 10$ and $W_u = 1000$, respectively. The time response of the error tracking for these two situations are presented in Fig. 6 and Fig. 7, respectively.

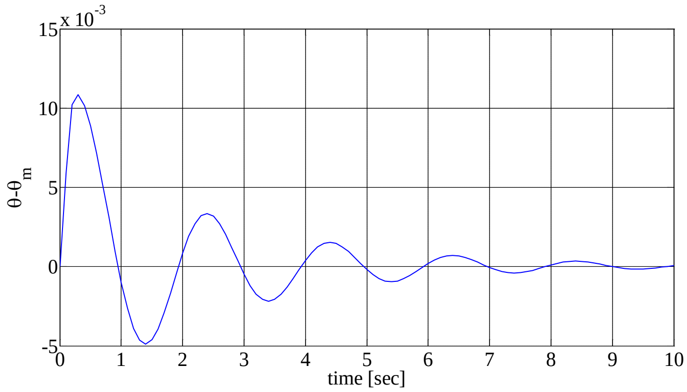

FIG. 6. Tracking error time response for $W_u = 10$

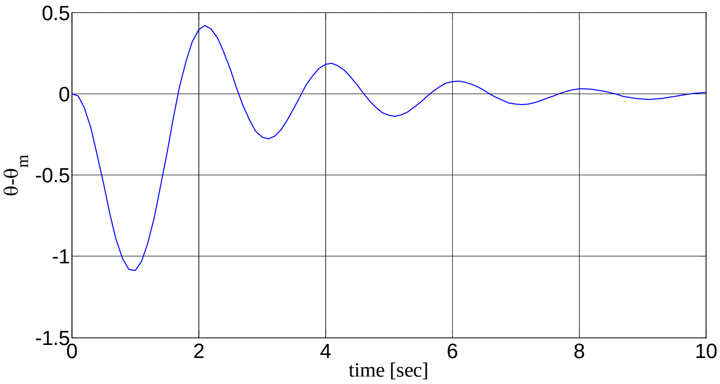

FIG. 7. Tracking error time response for $W_u = 1000$

The time response control δ_e for the two cases are given in Fig. 8 and Fig. 9, respectively.

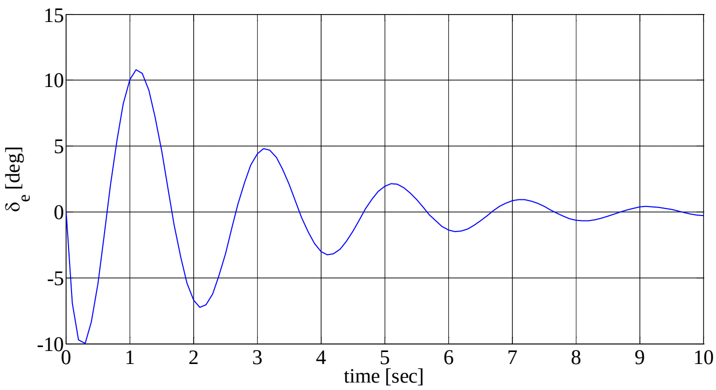

FIG. 8. Control variable δ_e time response for $W_u = 10$

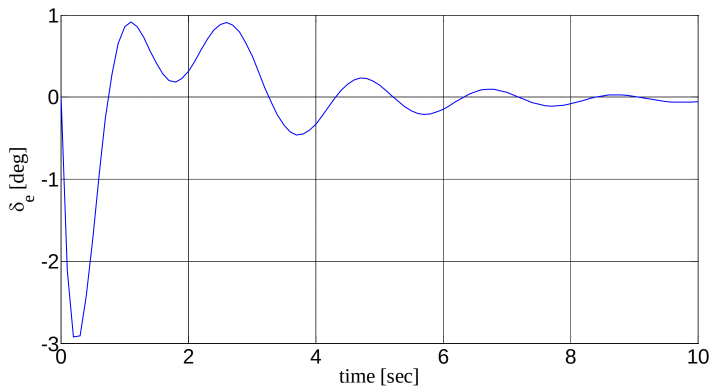

FIG. 9. Control variable δ_e time response for $W_u = 1000$

From the above plots it follows for both cases the settling time has been considerably reduced with respect to the case when the autopilot is not coupled. Moreover one also notices that when increasing the weighting W_u one obtains a smaller magnitude of the control variable but the tracking error of the ideal model response increases.

5. FINAL REMARKS

A modified loop-shaping H_∞ method with two degrees of freedom is presented for the design of the pitch attitude hold autopilot of an unmanned air vehicle. The numerical results and simulations indicate that the proposed approach is appropriate for acquiring robust stability, tracking of ideal model output and reduced sensitivity performances. These performances may be accomplished using a moderate complexity controller.

AKNOWLEDGMENT

This paper has been supported by MEN-UEFISCDI, Program Partnerships, Projects PN-II-PT-PCCA- 2013-4-1349 and PN-II-PT-PCCA- 2011-3.1-1560.

REFERENCES

[1] G. Stenfelt and U. Ringertz, Lateral Stability and Control of Tailless Aircraft Configuration, *Journal of Aircraft*, vol. 46, no. 6, pp. 2161-2164, 2009.

[2] A.D. Ngo, W.C. Reigelsperger and S.S. Banda, *Tailless Aircraft Control Law Design Using Dynamic Inversion and μ-Synthesis*, Proceedings of the 1996 IEEE International Conference on Control Applications, September 15-18, 1996, Dearborn, MI, pp. 107-112, 1996.

[3] M. Voskuijl, G. La Rocca and F. Dircken, *Controllability of Blended Wing Body Aircraft*, Proceedings of the 26th International Congress of the Aeronautical Sciences (ICAS), USA, 2008.

[4] R.W. Beard and T.W. Mclain, *Small Unmanned Aircraft: Theory and Practice*, Princeton University Press, 2012.

[5] R.J. Adams, J.M. Buffington, A.G. Sparks and S.S. Banda, Robust Multivariable Flight Control, Springer Verlag, 1994.

[6] B.L. Stevens and F.L. Lewis, *Aircraft Control and Simulation*, Wiley-Interscience,1992.

[7] A.-M. Stoica: *L_1 Controller Design for a Flying Wing Unmanned Aerial Vehicle*, Proceedings of ICMERA, 24-27 October 2013, Bucharest, Romania, 2013.

[8] V.G. Nair, M.V. Dileep and V.I. George, Aircraft yaw control system using LQR and fuzzy logic controller, *International Journal of Computer Applications*, vol. 45, no. 9, pp. 25-30, 2012.

[9] D. McFarlane and K. Glover, A loop shaping design procedure using H_∞ synthesis, *IEEE Transactions on Automatic Control*, vol. 37, no. 6, pp. 759-769, 1992.

[10] G. Papageorgiou, K. Glover, A. Smerlas and I. Poslethwaite, H_∞ Loop Shaping in *Robust Flight Control. A Design Challenge*, edited by J.F. Magni, S. Bennani and J. Terlouw, Springer-Verlag, pp. 64-80, 1997.

[11] G. Papageorgiou, K. Glover and R. A. Hyde, The H_∞ Loop Shaping Approach, in *Robust Flight Control. A Design Challenge*, edited by J.F. Magni, S. Bennani and J. Terlouw, Springer-Verlag, pp. 464-483, 1997.

[12] J. Lopez, R. Dormido, S. Dormido and J.P. Gomez, A Robust H_∞ Controller for an UAV Flight Control System, *Scientific World Journal, Hindawi Publishing Corporation*, Article ID 403236, 2015.

[13] User Guide for mini UAS Hirrus v1.2, Autonomous Flight Technology, 2015.

[14] D. McLean, *Automatic Flight Control Systems*, Prentice Hall International, 1990.

[15] D.C. McFarlane and K. Glover, *Robust Controller Design Using Normalized Coprime Factor Plant Descriptions*, Springer-Verlag, 1990.

[16] K. Glover, All optimal Hankel Norm Approximations of linear multivariable systems and their L_∞ -error bounds, *International Journal of Control*, Vol. 39, pp. 1115-1193, 1984.

[17] MIL-STD-1797A. *Flying Qualities of Piloted Aircraft*, 1997.

REMOTELY AND PILOTED AIRCRAFT SYSTEMS / LAW AND POLICIES