ABOUT GENERAL CONFORMAL ALMOST SYMPLECTIC N-LINEAR CONNECTIONS ON K-COTANGENT BUNDLE

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Abstract: *In the present paper starting from the notions of: almost symplectic structure and conformal almost symplectic structure, we define on k-cotangent bundle the notions of: conformal almost symplectic N-linear connection and general conformal almost symplectic N-linear connection. We determine the set of all general conformal almost symplectic N-linear connections in the case when the nonlinear connection is arbitrary and we find important particular cases.*

Keywords: *k-cotangent bundle, almost symplectic structure, conformal almost symplectic structure, conformal almost symplectic N-linear connection, general conformal almost symplectic N-linear connection.*

1. INTRODUCTION

The notion of Hamilton space was introduced by R. Miron in $[6]-[8]$. . The Hamilton spaces appear as dual via Legendre transformation, of the Lagrange spaces.

The differential geometry of the second order cotangent bundle was introduced and studied by R. Miron in [12] , R. Miron, D. Hrimiuc, H. Shimada, V.S. Sabău in [11], Gh.Atanasiu and M. Târnoveanu in [1], etc.

The differential geometry of the $k-$cotangent bundle was introduced and studied by R. Miron [10], [12].

In the present section we keep the general setting from R. Miron $[12]$, and subsequently we recall only some needed notions. For more details see $[12]$.

Let M be a real $n-$dimensional $C^{\infty}-$manifold and let $\left(T^{*k}M,\pi^{*k},M\right)$, $\left(k\geq 2,k\in N\right)$ be the $k-$cotangent bundle, where the total space is:

$$T^{*k}M=T^{*k-1}M\times T^{*}M. \tag{1}$$

Let $\left(x^{i},y^{(1)i},...,y^{(k-1)i},p_{i}\right),\left(i=1,2,...,n\right)$, be the local coordinates of a point $u=\left(x,y^{(1)},...,y^{(k-1)},p\right)\in T^{*k}M$ in a local chart on $T^{*k}M$.

We denote by:

$\widetilde{T}^{*k}M=T^{*k}M-\{0\}$ where $0:M\rightarrow T^{*k}M$ is the null section of the projection π^{*k}.

A change of local coordinates on the manifold $T^{*k}M$ is given by:

$$\begin{cases} \widetilde{x}^i = \widetilde{x}^i\left(x^1,...,x^n\right), \det\left(\dfrac{\partial \widetilde{x}^i}{\partial x^j}\right) \neq 0, \\ \widetilde{y}^{(1)i} = \dfrac{\partial \widetilde{x}^i}{\partial x^j} y^{(1)j}, \\ \text{..................................} \\ (k-1)\widetilde{y}^{(k-1)i} = \dfrac{\partial \widetilde{y}^{(k-2)i}}{\partial x^j} y^{(1)j} + ... + (k-1)\dfrac{\partial \widetilde{y}^{(k-2)i}}{\partial y^{(k-2)j}} y^{(k-1)j}, \\ \widetilde{p}_i = \dfrac{\partial x^j}{\partial \widetilde{x}^i} p_j, \end{cases} \tag{2}$$

We denote with N a nonlinear connection on the manifold $T^{*k}M, (k \geq 2, k \in N)$, with the coefficients:

$$\left(\underset{(1)}{N}{}^j{}_i\left(x, y^{(1)},..., y^{(k-1)}, p\right),..., \underset{(k-1)}{N}{}^j{}_i\left(x, y^{(1)},..., y^{(k-1)}, p\right), \right. \tag{3}$$

$$\left. N_{ij}\left(x, y^{(1)},..., y^{(k-1)}, p\right)\right), (i, j = 1,2,...,n).$$

The tangent space of $T^{*k}M$ in the point $u \in T^{*k}M$ is given by the direct sum of vector spaces:

$$T_u\left(T^{*k}M\right) = N_{0,u} \oplus N_{1,u} \oplus ... \oplus N_{k-2,u} \oplus V_{k-1,u} \oplus W_{k,u}, \forall u \in T^{*k}M \tag{4}$$

A local adapted basis to the direct decomposition (4) is given by:

$$\left\{ \frac{\delta}{\delta x^i}, \frac{\delta}{\delta y^{(1)i}},..., \frac{\delta}{\delta y^{(k-1)i}}, \frac{\delta}{\delta p_i} \right\}, (i = 1,2,...,n), \tag{5}$$

where:
$$\begin{cases} \dfrac{\delta}{\delta x^i} = \dfrac{\partial}{\partial x^i} - \underset{(1)}{N}{}^j{}_i \dfrac{\partial}{\partial y^{(1)j}} - ... - \underset{(k-1)}{N}{}^j{}_i \dfrac{\partial}{\partial y^{(k-1)j}} + N_{ij} \dfrac{\partial}{\partial p_j}, \\ \dfrac{\delta}{\delta y^{(1)i}} = \dfrac{\partial}{\partial y^{(1)}{}_i} - \underset{(1)}{N}{}^j{}_i \dfrac{\partial}{\partial y^{(2)j}} - ... - \underset{(k-2)}{N}{}^j{}_i \dfrac{\partial}{\partial y^{(k-1)j}}, \\ \text{..} \\ \dfrac{\delta}{\delta y^{(k-1)i}} = \dfrac{\partial}{\partial y^{(k-1)i}}, \\ \dfrac{\delta}{\delta p_i} = \dfrac{\partial}{\partial p_i} \end{cases} \tag{6}$$

and its dual basis $\{\delta x^i, \delta y^{(1)i}, \ \delta y^{(k-1)i}, \delta p_i\}$ determined by N and by the distribution W_k.

2. CONFORMAL ALMOST SYMPLECTIC STRUCTURE

Let D be an $N-$linear connection on $T^{*k}M$, with the local coefficients in the adapted basis (5):

$$D\Gamma(N)=\left(H^{i}{}_{jh},\underset{(\alpha)}{C}{}^{i}{}_{jh},C_{i}{}^{jh}\right),(\alpha=1,...,k-1). \tag{7}$$

D determines the $h-$, w_1-, w_2-,...,w_{k-1} covariant derivatives in the tensor algebra of d-tensor fields.

We consider on $\widetilde{T}^{*k}M$, $(k\geq 2,k\in N)$, an almost symplectic structure A given only by a nonsingular and skewsymmetric d-tensor field a_{ij}, of the type (0, 2) :

$$\begin{aligned}&A\left(x^{i},y^{(1)i},...,y^{(k-1)i},p_{i}\right)=\tfrac{1}{2}a_{ij}\left(x^{i},y^{(1)i},...,y^{(k-1)i},p_{i}\right)dx^{i}\wedge dx^{j}+\\&+a_{ij}\left(x^{i},y^{(1)i},...,y^{(k-1)i},p_{i}\right)dy^{(1)i}\wedge dy^{(1)j}+\tfrac{1}{2}a_{ij}\left(x^{i},y^{(1)i},...,y^{(k-1)i},p_{i}\right)\delta p_{i}\wedge\delta p_{j},\\&(i,j=1,2,...,n)\end{aligned} \tag{8}$$

The contravariant tensor field a^{ij} is obtained from the equations:

$a_{ij}a^{jk}=\delta_{i}^{k}$

Definition 1 An N−linear connection D is called almost symplectic if:

$$a^{ij}{}_{|h}=0,\ a_{ij}\overset{(\alpha)}{|}_{h}=0,\ a^{ij}\,|^{h}=0,(\alpha=1,...,k-1). \tag{9}$$

We associate to the lift A the operators of Obata's type given by:

$$\Omega_{hk}^{ij}=\frac{1}{2}(\delta_{h}^{i}\delta_{k}^{j}-a_{hk}a^{ij}),\ \Omega_{hk}^{*ij}=\frac{1}{2}(\delta_{h}^{i}\delta_{k}^{j}+a_{hk}a^{ij}). \tag{10}$$

Let $A_2(\widetilde{T}^{*k}M)$ be the set of all skewsymmetric d-tensor fields, of the type $(0,2)$ on $\widetilde{T}^{*k}M$ $k\geq 2,k\in N$. As is easily shown, the relations for $a_{ij},b_{ij}\in A_2(\widetilde{T}^{*k}M)$ defined by:

$$\begin{aligned}&(a_{ij}\approx b_{ij})\Leftrightarrow((\exists)\lambda(x,y^{(1)},...,y^{(k-1)},p)\in F(\widetilde{T}^{*k}M),\\&a_{ij}(x,y^{(1)},...,y^{(k-1)},p)=e^{2\lambda(x,y^{(1)},...,y^{(k-1)},p)}b_{ij}(x,y^{(1)},...,y^{(k-1)},p))\end{aligned} \tag{11}$$

is an equivalence relation on $A_2(\widetilde{T}^{*k}M)$.

Definition 2 The equivalent class $\hat{A}$ of $A_2(\widetilde{T}^{*k}M)/\approx$ to which A belongs, is called conformal almost symplectic structure on $T^{*k}M$.

Thus:

$$\begin{aligned}&\hat{A}=\{A'\mid a'_{ij}(x,y^{(1)},...,y^{(k-1)},p)=e^{2\lambda(x,y^{(1)},...,y^{(k-1)},p)}a_{ij}(x,y^{(1)},...,y^{(k-1)},p),\\&\lambda(x,y^{(1)},...,y^{(k-1)},p)\in F(\widetilde{T}^{*k}M)\}.\end{aligned} \tag{12}$$

3. GENERAL CONFORMAL ALMOST SYMPLECTIC N-LINEAR CONNECTIONS

Definition 3 An N-linear connection, D, with local coefficients: $D\Gamma(N) = \left(H^i{}_{jh}, \underset{(\alpha)}{C}{}^i{}_{jh}, C_i{}^{jh}\right)$, $(\alpha = 1,...,k-1)$, is called general conformal almost symplectic N-linear connection with respect to $\hat{A}$ if:

$$a_{ij|h} = K_{ijh},\ a_{ij} \overset{(\alpha)}{|}_h = \underset{(\alpha)}{Q}{}_{ijh},\ a_{ij}|^h = \dot{Q}_{ij}{}^h, \tag{13}$$

where $_{|h}, \overset{(\alpha)}{|}_h$ *and* $|^h$, denote the $h-, v_\alpha-$ and w_k- covariant derivatives with respect to D and $K_{ijh}, \underset{(\alpha)}{Q}{}_{ijh}, \dot{Q}_{ij}{}^h$ are arbitrary tensor fields on $T^{*^k}M$ of the types (0,3), (0,3) and (2,1) respectively, with the properties:

$$K_{ijh} = K_{jih}, \underset{(\alpha)}{Q}{}_{ijh} = \underset{(\alpha)}{Q}{}_{jih}, \dot{Q}_{ij}{}^h = \dot{Q}_{ji}{}^h, (\alpha = 1,...,k-1). \tag{14}$$

Definition 4 An N-linear connection, D, with local coefficients: $D\Gamma(N) = \left(H^i{}_{jh}, \underset{(\alpha)}{C}{}^i{}_{jh}, C_i{}^{jh}\right)$, $(\alpha = 1,...,k-1)$, for which there exists the 1-form ω, $\omega = \omega_i dx^i + \underset{(1)}{\dot{\omega}_i}\,\delta y^{(1)i} + ... + \underset{(k-1)}{\dot{\omega}_i}\,\delta y^{(k-1)i} + \ddot{\omega}^i \delta p_i$, such that:

$$\begin{cases} a_{ij|h} = 2\omega_h g_{ij},\ a_{ij} \overset{(\alpha)}{|}_h = \underset{(\alpha)}{\dot{\omega}_h}\, a_{ij}, \\ a_{ij}|^h = 2\ddot{\omega}^h a_{ij}, \end{cases} \tag{15}$$

where $_{|h}, \overset{(\alpha)}{|}_h$ *and* $|^h$, denote the $h-, v_\alpha-$ and w_k- covariant derivatives with respect to D, $(\alpha = 1,...,k-1)$ is called conformal almost symplectic N-linear connection, with respect to the conformal almost symplectic d-structure $\hat{A}$, corresponding to the 1-form ω and it is denoted by: $D\Gamma(N,\omega)$.

We shall determine the set of all general conformal almost symplectic N-linear connections, with respect to $\hat{A}$.

Let $\overset{0}{D}\Gamma(N) = \left(\overset{0}{H}{}^i{}_{jh}, \underset{(\alpha)}{\overset{0}{C}}{}^i{}_{jh}, \overset{0}{C}{}_i{}^{jh}\right)$ $(\alpha = 1,...,k-1)$ be the local coefficients of a fixed N-linear connection $\overset{0}{D}$, where $(N^j{}_i(x, y^{(1)},..., y^{(k-1)}, p), \underset{(\alpha)}{N}{}_{ij}(x, y^{(1)},..., y^{(k-1)}, p))$, $(\alpha = 1,...,k-1), (i, j = 1,2,...,n)$ are the local coefficients of the nonlinear connection N.

Then any N-linear connection, D, with the local coefficients $D\Gamma(N)=\left(H^i{}_{jh},\underset{(\alpha)}{C}{}^i{}_{jh},C_i^{jh}\right),(\alpha=1,...,k-1)$, can be expressed in the form [13]:

$$\begin{cases}\overline{H}^i{}_{sj}=H^i{}_{sj}-B^i{}_{sj},\\ \underset{(\alpha)}{\overline{C}}{}^i{}_{sj}=\underset{(\alpha)}{C}{}^i{}_{sj}-\underset{(\alpha)}{D}{}^i{}_{sj},(\alpha=1,...,k-1),(k\geq 2,k\in N),\\ \overline{C}_s^{ij}=C_s^{ij}-D_s^{ij}.\end{cases}\tag{16}$$

Using the relations (13), (16) and the Theorem 1 given by R.Miron in ([5]) for the case of Finsler connections we obtain:

Theorem 2 Let $\overset{0}{D}$ be a given N -linear connection, with local coefficients $\overset{0}{D}\Gamma(N)=\left(\overset{0}{H}{}^i{}_{jh},\underset{(\alpha)}{\overset{0}{C}}{}^i{}_{jh},\overset{0}{C}{}_i^{jh}\right)$ $(\alpha=1,...,k-1)$. The set of all general conformal almost symplectic N-linear connections, with respect to $\hat{A}$, corresponding to the same nonlinear connection N, with local coefficients $D\Gamma(N)=\left(H^i{}_{jh},\underset{(\alpha)}{C}{}^i{}_{jh},C_i^{jh}\right),(\alpha=1,...,k-1)$ is given by:

$$\begin{cases}H^i{}_{jh}=\overset{0}{H}{}^i{}_{jh}+\frac{1}{2}a^{im}(a_{mj\overset{0}{|}h}-K_{mjh})+\Omega_{sj}^{ir}X^s{}_{rh},\\ \underset{(\alpha)}{C}{}^i{}_{jh}=\underset{(\alpha)}{\overset{0}{C}}{}^i{}_{jh}+\frac{1}{2}a^{im}(a_{mj}\overset{\overset{(\alpha)}{0}}{|}_h-\underset{(\alpha)}{Q}{}_{mjh})+\Omega_{sj}^{ir}\underset{(\alpha)}{Y}{}^s{}_{rh},(\alpha=1,...,k-1),\\ C_i^{jh}=\overset{0}{C}{}_i^{jh}+\frac{1}{2}a^{mj}(a_{mj}\overset{0}{|}^h-\dot{Q}_{mi}{}^h)+\Omega_{si}^{jr}Z_r^{sh},\end{cases}\tag{17}$$

Where $_{\overset{0}{|}h}$, $\overset{\overset{(\alpha)}{0}}{|}_h$, and $\overset{0}{|}^h$ denote the $h-,v_\alpha-$ and w_k- covariant derivatives with respect to $\overset{0}{D}$, $X^i{}_{jh},\underset{(\alpha)}{Y}{}^i{}_{jh},Z_i^{jh}$ are arbitrary d-tensor fields and $K_{ijh},\underset{(\alpha)}{Q}{}_{ijh},\dot{Q}_{ij}{}^h$ are arbitrary d-tensor fields of the types (0,3), (0,3) and (2,1) respectively, with the properties $K_{ijh}=K_{jih},\underset{(\alpha)}{Q}{}_{ijh}=\underset{(\alpha)}{Q}{}_{jih},\dot{Q}_{ij}{}^h=\dot{Q}_{ji}{}^h,(\alpha=1,...,k-1).$

Particular cases:

1. If we take $K_{ijh}=2\omega_h a_{ij},\underset{(\alpha)}{Q}{}_{ijh}=2\underset{(\alpha)}{\dot{\omega}_h}a_{ij},(\alpha=1,...,k-1),\dot{Q}_{ij}{}^h=2\ddot{\omega}^h a_{ij}$ in Theorem 2, we obtain:

Theorem 3 Let $\overset{0}{D}$ be a given N -linear connection, with local coefficients $\overset{0}{D}\Gamma(N)==\left(\overset{0}{H}{}^i{}_{jh},\underset{(\alpha)}{\overset{0}{C}}{}^i{}_{jh},\overset{0}{C}{}_i^{jh}\right)$ $(\alpha=1,...,k-1)$.

The set of all conformal almost symplectic N-linear connections with respect to $\hat{A}$, corresponding to the 1-form ω, with local coefficients $D\Gamma(N,\omega) = \left(H^i{}_{jh}, \underset{(\alpha)}{C}{}^i{}_{jh}, C_i^{jh}\right), (\alpha = 1,...,k-1)$ is given by:

$$\begin{cases} H^i{}_{jh} = \overset{0}{H}{}^i{}_{jh} + \frac{1}{2}a^{im}(a_{mj\overset{0}{|}h} - 2\omega_h a_{mj}) + \Omega^{ir}_{sj} X^s{}_{rh}, \\ \underset{(\alpha)}{C}{}^i{}_{jh} = \underset{(\alpha)}{\overset{0}{C}}{}^i{}_{jh} + \frac{1}{2}a^{im}(a_{mj} \overset{(\alpha)}{\overset{0}{|}}{}_h - 2\underset{(\alpha)}{\dot{\omega}_h} a_{mj}) + \Omega^{ir}_{sj} \underset{(\alpha)}{Y}{}^s{}_{rh}, (\alpha = 1,...,k-1), \\ C_i^{jh} = \overset{0}{C}{}_i^{jh} + \frac{1}{2}a^{mj}(a_{mi} \overset{0}{|}{}^h - 2\ddot{\omega}^h a_{mi}) + \Omega^{jr}_{si} Z_r^{sh}, (i,j,h = 1,2,...,n), \end{cases} \quad (18)$$

where $\overset{0}{|}h$, $\overset{(\alpha)}{\overset{0}{|}}{}_h$, and $\overset{0}{|}{}^h$ denote the $h-, v_\alpha -$ and $w_k -$ covariant derivatives with respect to $\overset{0}{D}$, $X^i{}_{jh}, \underset{(\alpha)}{Y}{}^i{}_{jh}, Z_i^{jh}$ are arbitrary d-tensor fields, $(\alpha = 1,...,k-1)$, $\omega = \omega_i dx^i + \underset{(1)}{\dot{\omega}_i} \delta y^{(1)i} + ... + \underset{(k-1)}{\dot{\omega}_i} \delta y^{(k-1)i} + \ddot{\omega}^i \delta p_i$, is an arbitrary 1-form and Ω is the operator of Obata's type given by (10).

2. If $X^i{}_{jh} = \underset{(\alpha)}{Y}{}^i{}_{jh} = Z_i^{jh} = 0$, in Theorem 2 we have:

Theorem 4 Let $\overset{0}{D}$ be a given N -linear connection, with local coefficients $\overset{0}{D}\Gamma(N) = \left(\overset{0}{H}{}^i{}_{jh}, \underset{(\alpha)}{\overset{0}{C}}{}^i{}_{jh}, \overset{0}{C}{}_i^{jh}\right)$ $(\alpha = 1,...,k-1)$. Then the following N-linear conection K, with local coefficients $K\Gamma(N) = \left(H^i{}_{jh}, \underset{(\alpha)}{C}{}^i{}_{jh}, C_i^{jh}\right), (\alpha = 1,...,k-1)$, given by (19) is general conformal almost symplectic with respect to $\hat{A}$:

$$\begin{cases} H^i{}_{jh} = \overset{0}{H}{}^i{}_{jh} + \frac{1}{2}a^{im}(a_{mj\overset{0}{|}h} - K_{mjh}), \\ \underset{(\alpha)}{C}{}^i{}_{jh} = \underset{(\alpha)}{\overset{0}{C}}{}^i{}_{jh} + \frac{1}{2}a^{im}(a_{mj} \overset{(\alpha)}{\overset{0}{|}}{}_h - \underset{(\alpha)}{Q}{}_{mjh}), (\alpha = 1,...,k-1), \\ C_i^{jh} = \overset{0}{C}{}_i^{jh} + \frac{1}{2}a^{mj}(a_{mi} \overset{0}{|}{}^h - \dot{Q}_{mi}{}^h), \end{cases} \quad (19)$$

where $\overset{0}{|}h$, $\overset{(\alpha)}{\overset{0}{|}}{}_h$, and $\overset{0}{|}{}^h$ denote the $h-, v_\alpha -$ and $w_k -$ covariant derivatives with respect to $\overset{0}{D}$, and $K_{ijh}, \underset{(\alpha)}{Q}{}_{ijh}, \dot{Q}_{ij}{}^h$ are arbitrary d-tensor fields of the types (0,3), (0,3) and (2,1) respectively, with the properties: $K_{ijh} = K_{jih}, \underset{(\alpha)}{Q}{}_{ijh} = \underset{(\alpha)}{Q}{}_{jih}, \dot{Q}_{ij}{}^h = \dot{Q}_{ji}{}^h, (\alpha = 1,...,k-1)$.

3. If we take a general conformal almost symplectic N-linear connection with respect to $\hat{A}$ as $\overset{0}{D}$, in Theorem 2 we have:

Theorem 5 Let $\overset{0}{D}$ be on $T^{*k}M$ a fixed general conformal almost symplectic N-linear connection with respect to $\hat{A}$, with the local coefficients $\overset{0}{D}\Gamma(N) = \left(\overset{0}{H}{}^{i}{}_{jh}, \underset{(\alpha)}{\overset{0}{C}}{}^{i}{}_{jh}, \overset{0}{C}{}_{i}^{jh} \right)$ $(\alpha = 1,..., \; k-1)$. The set of all general conformal almost symplectic N-linear connections, with respect to $\hat{A}$, with local coefficients $D\Gamma(N) = \left(H^{i}{}_{jh}, \underset{(\alpha)}{C}{}^{i}{}_{jh}, C_{i}^{jh} \right), (\alpha = 1,...,k-1)$ is given by:

$$\begin{cases} H^{i}{}_{jh} = \overset{0}{H}{}^{i}{}_{jh} + \Omega^{ir}_{sj} X^{s}{}_{rh}, \\ \underset{(\alpha)}{C}{}^{i}{}_{jh} = \underset{(\alpha)}{\overset{0}{C}}{}^{i}{}_{jh} + \Omega^{ir}_{sj} \underset{(\alpha)}{Y}{}^{s}{}_{rh}, (\alpha = 1,...,k-1), \\ C_{i}^{jh} = \overset{0}{C}{}_{i}^{jh} + \Omega^{jr}_{si} Z_{r}^{sh}, \end{cases} \tag{20}$$

where $X^{i}{}_{jh}, \underset{(\alpha)}{Y}{}^{i}{}_{jh}, Z_{i}^{jh}$ are arbitrary d-tensor fields, $(\alpha = 1,...,k-1)$.

4. If $K_{ijh} = \underset{(\alpha)}{Q}{}_{ijh} = \dot{Q}_{ij}{}^{h} = 0, (\alpha = 1,...,k-1)$ in Theorem 2 we obtain the set of all almost symplectic N-linear connection in the case when the nonlinear connection is fixed:

Theorem 6 Let $\overset{0}{D}$ be a given N -linear connection, with local coefficients $\overset{0}{D}\Gamma(N) = \left(\overset{0}{H}{}^{i}{}_{jh}, \underset{(\alpha)}{\overset{0}{C}}{}^{i}{}_{jh}, \overset{0}{C}{}_{i}^{jh} \right)$ $(\alpha = 1,...,k-1)$. The set of all almost symplectic N-linear connections, with respect to $\hat{A}$, corresponding to the same nonlinear connection N, with local coefficients $D\Gamma(N) = \left(H^{i}{}_{jh}, \underset{(\alpha)}{C}{}^{i}{}_{jh}, C_{i}^{jh} \right), (\alpha = 1,...,k-1)$ is given by:

$$\begin{cases} H^{i}{}_{jh} = \overset{0}{H}{}^{i}{}_{jh} + \frac{1}{2} a^{im} a_{mj \overset{0}{|} h} + \Omega^{ir}_{sj} X^{s}{}_{rh}, \\ \underset{(\alpha)}{C}{}^{i}{}_{jh} = \underset{(\alpha)}{\overset{0}{C}}{}^{i}{}_{jh} + \frac{1}{2} a^{im} a_{mj} \overset{(\alpha)}{\overset{0}{|}}{}_{h} + \Omega^{ir}_{sj} \underset{(\alpha)}{Y}{}^{s}{}_{rh}, (\alpha = 1,...,k-1), \\ C_{i}^{jh} = \overset{0}{C}{}_{i}^{jh} + \frac{1}{2} a^{mj} a_{mj} \overset{0}{|}{}^{h} + \Omega^{jr}_{si} Z_{r}^{sh}, \end{cases} \tag{21}$$

where $_{\overset{0}{|} h}$, $\overset{(\alpha)}{\overset{0}{|}}{}_{h}$, and $\overset{0}{|}{}^{h}$ denote the $h-, v_{\alpha}-$ and w_k- covariant derivatives with respect to $\overset{0}{D}$, $X^{i}{}_{jh}, \underset{(\alpha)}{Y}{}^{i}{}_{jh}, Z_{i}^{jh}$ are arbitrary d-tensor fields.

Theorem 7 *The mappings* $D\Gamma(N) \rightarrow \overline{D}\Gamma(N)$ *determined by* (20), *together with the composition of these mappings is an abelian group.*

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