MATHEMATICAL ASPECTS REGARDING THE BASIS OF DECISION MAKING IN MILITARY ACTIONS

Traian ANASTASIEI[*], Gabriel RĂDUCANU[**], Radu-Mihai DINCĂ[*]

[*]"Henri Coandă" Air Force Academy, Braşov, Romania (a_traian03@yahoo.com, radu_mihai1989@yahoo.com)
[**]Romanian Air Force Staff, Bucharest, Romania (gabiraducanu@yahoo.ca)

DOI: 10.19062/1842-9238.2017.15.2.5

Abstract: *Modern military actions cannot be conceived in the absence of reasonable scientific approach. The selection of the best course of action (CoA) for the achievement of targeted objectives, while taking into account the available resources and the configuration of the internal / external background, is an extremely complex activity that is carried out under conditions of uncertainty and time constraints. Optimization of military actions as well as the implementation of efficient decision-making solutions is a matter of prime importance in the current battlefield configuration and it implies a rigorous mathematical apparatus. In this article, we analyze the importance of detecting/adjusting mathematical methods in accordance with the new technologies and armaments used in contemporary confrontations, using a particular case in the field of aviation.*

Keywords: *decision-making, aviation, military actions, mathematical methods*

1. INTRODUCTION

The military phenomenon is one of the social phenomena that best illustrates the idea of development - the forces and the whole complex of Tactics, Techniques, Procedures (TTPs) are subjected to dramatic changes in the context of the fundamental processes of social evolution. The causes of transformations are multiple and diverse, corresponding to the nature of society itself, and they can be found in any of its components.

Decryption of changes and trends occurring at geopolitical and geostrategic level and the awareness of possible developments in the field of military actions, especially those which are conducted in the airspace, constitute actions which are meant to provide inputs for developing a robust, resilient and versatile air force able to act within a fluid and unpredictable security environment. Nowadays military actions take place in an increasingly complex operation area which usually involves a lot of entities (armed forces, different factions, local population, NGOs, civilian agencies, economic agents etc.), many of them having divergent interests. In order to succeed, decision-makers need to adopt a comprehensive approach and they must act quickly and "surgically". There is a very short time for analysis, evaluation and decision-making and there is no place for error. [4, 5, 6]

Modern warfare, regardless of its form, type or magnitude, cannot be conceived without the active presence of the air force (aviation, radars, artillery / SAMs etc.) capable of executing rapid, surprising and destructive strikes, of generating strong and long-lasting effects on the enemy's ability to wage war. It can also disrupt enemy's actions by stealth, maneuverability and precise actions.

Another element which is strongly connected with the air force is represented by the advanced technologies. Air power can be considered one of the finest products of technological revolution initiated at the dawn of the 20^{th} century. The intrinsic connection with technology allows air forces to exploit the benefits of outer space. The actions taken in a three-dimensional space provide air component with greater freedom of action than the other components (land and maritime).

Military actions are, by definition, planned, organized and conducted by military structures. Decision is the essence of military action management. In the complex and contradictory world in which we live, adopting and implementing high-performance decisions is increasingly difficult and at the same time necessary. [2] The greater our world's complexity and contradictions are, the more effective our decisions should be. Any mission aims to achieve a goal set by the higher echelon or resulting from the commander's plan.

2. ASPECTS REGARDING THE EFFECTIVENESS OF ACTIONS IN AVIATION

Generally speaking, the effectiveness of our actions obviously depends on the resources used and the quality of our decisions. In the broadest sense, this means that our ability to produce a certain effect on others' behavior (when we refer to human action, the effect is anticipated to become a *goal*) is based upon our ability to plan our future actions, to allot the needed resources and to act in accordance with our plan but also by taking into account the changes that occur in reality. We should take into account some restrictions or limitations: available resources, TTPs to use, time etc. Consequently, sometimes a normative and not a descriptive approach is needed, so formal methods and models are needed in order to set up your actions on scientific bases.

Most of the combat actions are designed to produce a desired level of attrition in the enemy's forces and logistics or to destroy them. Usually, many targets are located inside the enemy's territory (beyond the *Forward Edge of Battle Area - FEBA*) and this fact increases the uncertainty level regarding the final effects on target.

A course of action (CoA) which involves air strikes upon a major objective located in the depth of the enemy's territory implies the fact that the aircraft(s) will cross areas which pose different levels of hostility, as follows:

- the flight within territory controlled by own troops or allies ("the friendly area"), an area which has a very low probability of danger, basically 0 %;

- the flight within territory controlled by enemy forces ("the hostile area"), an area which comprises many Surface Based Air Defence (SBAD) threats, so the probability of having losses is getting higher;

- the flight within the district where the target is located ("the dangerous area"), an area which is strongly defended by SBAD systems, so this area has a high level of risk, and the probability of getting losses cannot be neglected (we should also notice that this phase includes preparatory maneuvers and the air strike itself).

If we analyze the losses suffered by our own forces in every phase of the mission we shall conclude that it is unprofitable, from the resource perspective, to get the aircraft to the target and not to use the load on board. [1] Friendly air forces may suffer certain losses caused by the enemy's reaction, which is carried out either with its own air forces or by GBAD means (SAMs or anti-aircraft artillery - AAA). Considering this, we can deduce a probability of reaching the target which is represented by the following relations:

$$Q = Q_{SAM} * Q_{AAA} * Q_{AF} \quad (1)$$

where:
Q - is the probability that the a/c will encounter the target and will launch the strike against it;
Q_{SAM} - is the probability that the a/c will get into the district where the target is located taking into account the SAMs reaction;
Q_{AAA} - is the probability that the a/c will get into the district where the target is located taking into account the antiaircraft artillery reaction;
Q_{AF} - is the probability that the a/c will get into the district where the target is located taking into account the enemy air forces reaction.

In order to establish which variables are relevant in the decision-making process, the air force staff must establish certain criteria for evaluating the strikes' efficiency. Based on these criteria the staff proceeds to a cost/benefit analysis or a similar method and then it will design the best CoAs. The commander will select the one which best fits his intention. Speaking of criteria, the most used criteria are:

- the amount of damage produced to the target (percentage and quantitative);
- the amount of resources (number of aircraft, manpower, fuel etc.) necessary to fulfill the mission.

Papers that deal with this topic provide methods of calculation for some of these criteria. [3]

a) The average percentage of losses incurred by the enemy when acting against it with more than one aircraft.

a1) When the attack is carried out by aiming the constituent elements of the group of targets, the average percentage of enemy's losses, denoted by *W**, is calculated by the following equation:

$$W^* = 1 - e^{-\frac{N}{N_t} * W * N_{at} * Q} \quad (2)$$

where:
N - the number of aircraft performing the attack on the group of targets;
N_t - the number of targets (elementary objectives) within the group of targets;
W - the probability that an aircraft destroys, throughout a single attack, an element within the group of targets. This variable depends on the target's features (dimensions, degree of vulnerability) and the features of the ammunition used;
N_{at} - The number of attacks carried out by each aircraft;
Q - The probability that the aircraft will reach the target district in order to execute the attack. It depends on the enemy's retaliation means (air force, artillery/anti-aircraft missiles etc.). It is determined by the (1) equation.

a2) When the attack is conducted against a group of targets considered as a whole, so that the elements located in the vulnerable zone of the group of targets are considered to be destroyed, while the others are not, the average percentage of losses, denoted by M_N, is calculated by the relation:

$$M_N = 1 - (1 - M)^N \quad (3)$$

where:
N - The number of aircraft performing the attack on the group of targets;
M - The average percentage of damage when the group of targets is attacked by a single aircraft;

$$M = M_x * M_y \quad (4)$$

where:
M_x, M_y - mean values of the coverage percentage determined by area of destruction (first-order first moment), calculated as follows:

$$M_x = \frac{E_x}{T_x}\left[x_1 * \Phi(x_1) - x_2 * \Phi(x_2) + \frac{1}{\rho\sqrt{\pi}} * (e^{-\rho^2 * x_1^2} - e^{-\rho^2 * x_2^2})\right];$$

$$M_y = \frac{E_y}{T_y}\left[y_1 * \Phi(y_1) - x_2 * \Phi(y_2) + \frac{1}{\rho\sqrt{\pi}} * (e^{-\rho^2 * y_1^2} - e^{-\rho^2 * y_2^2})\right];$$

with:

$$x_1 = \frac{T_x + L_x}{2 * E_x};$$

$$x_2 = \frac{T_x - L_x}{2 * E_x};$$

$$y_1 = \frac{T_y + L_y}{2 * E_y};$$

$$y_2 = \frac{T_y + L_y}{2 * E_y};$$

where:

T_x, T_y - the dimensions of objective (target), approximated by a rectangle;

L_x, L_y - the dimensions of the area of destruction, approximated by a rectangle;

E_x, E_y - The probable deviations (probable error) of the center of the area of destruction relative to the center of the objective (target);

$\Phi(x)$ - the Laplace transformed function;

ρ - 0,476936.

b) The required number of aircraft in order to achieve an average predetermined percentage of losses for a group of targets, denoted by N, is calculated as follows:

b1) When the attack is carried out by aiming the constituent elements of the group of targets, we use the equation:

$$N = -\frac{N_t * \ln(1 - P_1)}{N_{at} * W * Q} \quad (5)$$

where:

P_1 - The predetermined average percentage of the damage caused to the objective (target);
N_t - The number of targets (elementary objectives) within the group of targets;
W - the probability that an aircraft destroys, on a single attack, an element within the group of targets;
N_{at} - The number of attacks carried out by each aircraft;
Q - As it was shown above.

b2) When the attack is conducted against a group of targets considered as a whole, we use:

$$N = \frac{\ln(1 - P_1)}{\ln(1 - M)} \quad (6)$$

where:

P_1 - The predetermined average percentage of the damage caused to the objective (target);
M - The average percentage of damages when the group of targets is attacked by a single aircraft.

3. AVIATION IN COMPLEX ACTIONS

Modern aircraft are characterized by advanced fighting capabilities, including fire power, stealth technologies, radar and electronic warfare (EW) equipment and above all, the technical and tactical performances which allows it to carry out complex actions in a complex environment. In order to determine some efficiency indicators, we will use a hypothetical situation:

3.1 Problem formulation

In a district (hereinafter referred to as **objective**) there are several constituent elements (hereinafter referred to as **targets**). The target is attacked by several aircraft, each of which is equipped with several types of weaponry (combat cargo).

For tactical (objective and target characteristics, attack procedure) and technical (launch / firing parameters) reasons, the battle load will not be launched all at once, the aircraft repeating the attack. It is required to determine **the main efficiency indicators** (average damage caused to the enemy, the number of aircraft required to carry out the mission).

3.2 Working hypotheses

In order to compute the calculations, we set out some simplifying hypotheses that will not influence the final result:

1. We consider that the targets are evenly distributed within the objective, which means that in a Z area (representing z % of S - the total area of the objective) there will be a number of T targets (representing z % of NT - total number of targets within the objective), so T is proportional to Z.

2. We consider that the attack will be carried out by means of 2 types of weapons / battle load:

- Type 1 produces effects on a large surface (all targets in the "vulnerable" area are destroyed);
- Type 2 produces point effects (only the directly hit target is destroyed).

From the probabilistic point of view we define:

- E_i - the event consisting of hitting and damaging the target, i.e. the destruction of the appropriate targets, using the type of weapon / battle load;
- $P(E_i)$ - the probability of producing E_i event.

3. We solve the problem using the above mentioned equations:

- for Type 1 weapons - equations (2) and (5);
- for Type 2 weapons - equations (3), (4) and (6).

We also add the following points:

- each aircraft performs successive attacks, each attack is carried out with a single type of weaponry;
- during the first attacks, type 1 weapons are used, but the following ones are carried out with type 2 weapons;
- events E_i are compatible, meaning that targets are destroyed whether they are hit by Type 1 weapons or by Type 2 weapons.

3.3 Solution

Whether or not the results of the attack are known immediately after the use of a specific type of weapons determines two situations that we will analyze in turn.

Case I - The results of the attack cannot be immediately known (the destroyed targets cannot be identified), so the following attacks can also be directed against targets that have already been hit.

In this case, events E_1 and E_2 are independent, meaning that even if a target was destroyed during the first attack, this does not mean that there will be no further attack on it. Thus,

$$P(E_1, E_2) = P(E_1) + P(E_2) - P(E_1) * P(E_2) \tag{7}$$

where:
P (E_1, E_2) - The probability of destroying the target using both types of weapons.
Considering the above and noting P (E_1, E_2) with P_p, it results:

$$P_p = M_N + W^* - M_N * W^* \tag{8}$$

By substituting and making calculations, based on the above equations, we obtain:

$$P_p = M_N + W^* - M_N * W^* =$$
$$= M_N + W^*(1 - M_N) =$$
$$= 1 - (1 - M)^N + W^*(1 - (1 - (1 - M)^N)) =$$
$$= 1 - (1 - M)^N + W^*(1 - M)^N =$$
$$= 1 - (1 - M)^N * (1 - W^*) =$$
$$= 1 - (1 - M)^N * (1 - (1 - e^{-\frac{N}{N_t} * W * N_{at} * Q})) =$$
$$= 1 - (1 - M)^N * e^{-\frac{N}{N_t} * W * N_{at} * Q} =$$
$$= 1 - ((1 - M) * e^{-\frac{W * N_{at} * Q}{N_t}})^N$$

We thus obtained a relation to determine the average percentage of enemy's losses when two types of armaments (Type 1 and Type 2) are used:

$$P_p = 1 - (1 - M)^N * e^{-\frac{N}{N_t} * W * N_{at} * Q} \tag{9}$$

The **average number of destroyed targets** is then determined by:

$$M_t = N_t * P_p \tag{10}$$

From equation (9) we obtain:

$$1 - P_p = ((1 - M) * e^A)^N,$$

where $A = -\frac{W * N_{at} * Q}{N_t}$

Then, by applying the logarithm,

$$\ln(1 - P_p) = \ln((1 - M) * e^A)^N =$$
$$= N * \ln((1 - M) * e^A) =$$
$$= N * (\ln(1 - M) + \ln e^A) =$$
$$= N * (\ln(1 - M) + A)$$

Thus, we obtain the N - the required number of aircraft in order to produce P_p average losses to the enemy.

N = $\frac{\ln(1-P_p)}{\ln(1-M)+A}$, or

$$N = \frac{\ln(1-P_p)}{\ln(1-M)-\frac{W*N_{at}*Q}{N_t}} \quad (11)$$

Case II - The conditions allow us to find out the outcome of the attack immediately, we can identify the destroyed targets, so the following attacks will be directed only on targets that have not yet been destroyed.

Identification of the main efficiency indicators will be done by means of the following algorithm:

- we calculate the average number of targets destroyed as a result of the attack carried out with Type 1 weapons (denoted by $M^{(1)}$). For the symmetry of the notations, we will note with $N^{(1)}$) - the initial number of targets (previously noted with N_t);

$$M_t^{(1)} = M_N * N_t^{(1)} \quad (12)$$

- we calculate the mean number of targets that have not been destroyed after the attack was carried out with Type 1 weapons (denoted by $N^{(2)}$). For these targets there will be used the Type 2 weapons.

$$N_t^{(2)} = N_t^{(1)} - M_t^{(1)} =$$

$$= N_t^{(1)} - M_N * N_t^{(1)} =$$

$$= N_t^{(1)} * (1 - M_N) =$$

$$= N_t^{(1)} * (1 - (1-(1 - M)^N)) =$$

$$= N_t^{(1)} * (1 - M)^N$$

$$N_t^{(2)} = N_t^{(1)} * (1 - M)^N \quad (13)$$

- we calculate the average number of targets that were not destroyed after the attack carried out with Type 2 weapons (denoted by $M^{(2)}$);

$$M_t^{(2)} = N_t^{(2)} * W^* \quad (14)$$

- we calculate the average number of targets destroyed after the final attack (denoted M_t);

$$M_t = M_t^{(1)} + M_t^{(2)} \quad (15)$$

Using relations (12), (14) and (15), it results:

$$M_t = M_N * N_t^{(1)} + N_t^{(2)} * W^* =$$

$$= M_N * N_t^{(1)} + N_t^{(1)} * (1 - M)^N * W^* =$$

$$= N_t^{(1)} * (1 - (1 - M)^N + (1 - M)^N * W^*) =$$

$$= N_t^{(1)} * (1 - (1 - M)^N * (1 - W^*)) =$$

$$= N_t^{(1)} * (1 - (1 - M)^N * (1 - (1 - e^{-\frac{N}{N_t^{(2)}} * W * N_{at} * Q}))) =$$

$$= N_t^{(1)} * (1 - (1 - M)^N * e^{-\frac{N}{N_t^{(2)}} * W * N_{at} * Q})$$

$$M_t = N_t^{(1)} * (1 - (1 - M)^N * e^{-\frac{N}{N_t^{(1)} * (1-M)^N} * W * N_{at} * Q}) \qquad (16)$$

We divide both members of this last relation by $N_t^{(1)}$). But M_t / $N_t^{(1)}$) = P_p. We obtain successively:

$$P_p = 1 - (1 - M)^N * e^{-\frac{N}{N_t^{(2)}} * W * N_{at} * Q}$$

$$1 - P_p = (1 - M)^N * e^{-\frac{N}{N_t^{(2)}} * W * N_{at} * Q}$$

$$\ln(1 - P_p) = \ln((1 - M)^N * e^{-\frac{N}{N_t^{(2)}} * W * N_{at} * Q})$$

$$\ln(1 - P_p) = \ln(1 - M)^N + \ln e^{-\frac{N}{N_t^{(2)}} * W * N_{at} * Q}$$

$$\ln(1 - P_p) = \mathrm{N} * \ln(1 - M) - \frac{N}{N_t^{(2)}} * W * N_{at} * Q$$

Dividing by N and replacing $N_t^{(2)}$ with $N_t^{(1)} * (1 - M)^N$(equation (13), we obtain:

$$\frac{\ln(1 - P_p)}{\mathrm{N}} - \ln(1 - M) = \frac{-W * N_{at} * Q}{N_t^{(1)} * (1 - M)^N} \qquad (17)$$

Since this equation is more difficult to solve, we will form 2 functions of variable N:

$$f_1(N) = \frac{\ln(1 - P_p)}{\mathrm{N}} - \ln(1 - M) \qquad (18)$$

$$f_2(N) = \frac{-W * N_{at} * Q}{N_t^{(1)} * (1 - M)^N} \qquad (19)$$

Finding N can be done either by the graphical method, at the intersection of the graphs of the two functions *f1(N)* and *f2(N)*, or by numerical methods, using the computer. We used both methods and the results are highlighted in *Figure 1*. For the numerical determination of *N*, we considered 4) case from *Table 1* and we used the *Maple 13* software. This software also helped us to provide the graphs for the functions.

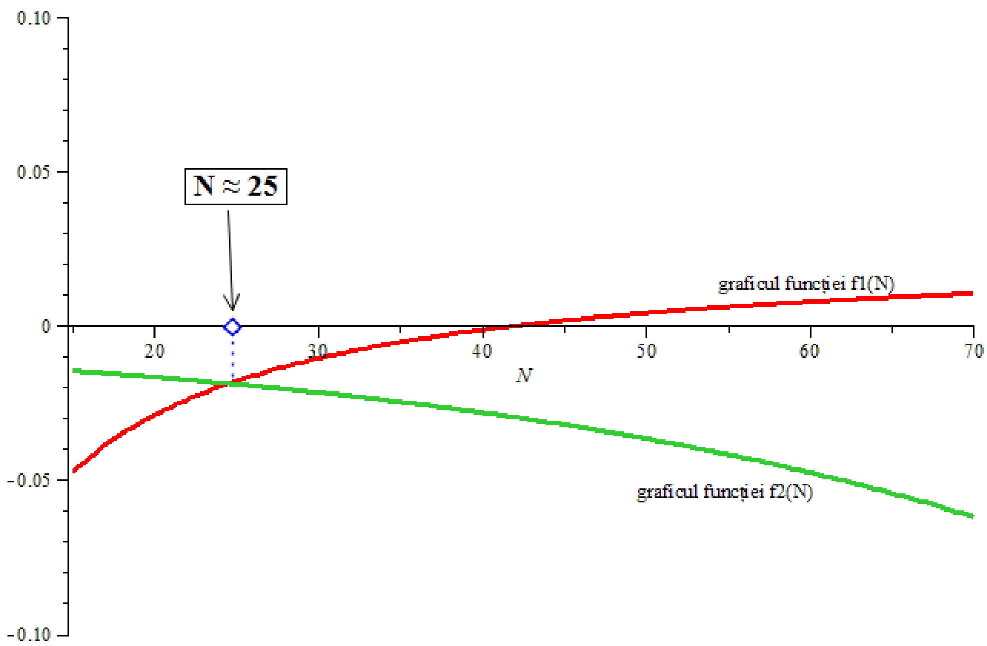

FIG. 1. Finding N by graphic method

Example (simplified, without affecting the final results)

There are 100 tanks in a district. We have to determine the required number of aircraft in order to destroy 2/3 (67) of them using 2 types of armament.

The results are shown in Table 1 (Pay attention to the order of magnitude!):

Table 1. The required number of aircrafts in certain conditions

The type of weapons used	The required number of aircrafts
1) Type I weapons (ammunition)	42
2) Type II weapons (ammunition)	112
3) Type I and II weapons (ammunition), without "fire transport"	31
4) Type I and II weapons (ammunition), with " fire transport "	25

CONCLUSIONS

The evolution of the air force is strongly related to the technological and scientific development as well as to the evolution of the regional and global security environment. Accordingly, the future of air force can only be designed in the context of the European and Euro-Atlantic security system. In such a system there are many interdependencies which are often difficult to quantify. It is the duty of military specialists to study, analyze and propose measures to decision-makers to effectively employ the air force in future operations.

Military actions are dynamic, and each mission is different and unrepeatable. In turn, advanced technologies radically change the means of military action. All of these require that decision makers use appropriate methods (simulation, modeling, conceptualization etc.) in order to base the decisions they make on a scientific foundation.

With regard to establishing the optimal need for aviation forces, we have found that the existing manuals / regulations contain methodologies that do not solve all the possible situations.

In this paper we have obtained a system of relations that allows us to find out the solution to the formulated problem, as follows:

- the percentage of average losses produced to the objective is given by relation (9);
- the average number of destroyed targets is given in **Case I** (without the immediate identification of damaged targets) by the relation (10), and in **Case II** (only the previously undamaged targets are attacked) by relation (16);
- the number of aircraft required to produce certain predetermined losses to the objective is presented in **Case I** by relation (11), and in **Case II** by relation (17).

REFERENCES

[1] Traian Anastasiei, *Aviaţia – alonja acţiunilor militare*, Editura Academiei Forţelor Aeriene „Henri Coandă", Braşov, 2002, pp. 170-173;

[2] Ovidiu Nicolaescu, *Sistemul decizional al organizaţiei*, Editura Economică, Bucureşti, 1998, p. 17;

[3] Valerian Nestorescu, *Calculul probabilităților în domeniul militar*, Editura Militară, București, 1963, pp. 140-214;

[4] ***Joint Air Power Competence Centre, *Present Paradox – Future Challenge,* March 2014, available at: https://www.japcc.org/portfolio/present-paradox-future-challenge/, accessed on 12 May 2017;

[5] ***Joint Air Power Competence Centre, *Air & Space Power in NATO – Future Vector Part I,* July 2014, available at: https://www.japcc.org/portfolio/air-space-power-in-nato-future-vector-part-i/, accessed on 12 May 2017;

[6] ***Joint Air Power Competence Centre, *Air & Space Power in NATO – Future Vector Part II,* October 2014, available at: https://www.japcc.org/portfolio/air-space-power-in-nato-future-vector-part-ii/, accessed on 12 May 2017.